Course 2BA1: Michaelmas Term 2004.

Assignment II.

To be handed in by Friday 14th January, 2005.
Please include both name and student number on any work handed in.

1. For each of the following relations, determine whether or not that relation is reflexive, symmetric, transitive, anti-symmetric, an equivalence relation, and/or a partial order, giving appropriate reasons for your answers:—

   (i) the relation P on the set $\mathbb{R}$ of real numbers, where real numbers x and y satisfy xPy if and only if $x^3 - y^3 + x - y \geq 0$.

   (ii) the relation Q on the set $\mathbb{R}$ of real numbers, where real numbers x and y satisfy xQy if and only if $x^3 - y^3 + x - y = k$ for some integer k.

   (iii) the relation R on the set $\mathbb{R}$ of real numbers, where real numbers x and y satisfy xRy if and only if $xy \geq 0$.

2. For each of the following functions, determine whether or not that function is injective and/or surjective, and whether or not it has a well-defined inverse, giving appropriate reasons for your answers:—

   (i) the function $f: [1, 3] \to [1, 2]$ with $f(x) = 4x - x^2 - 2$ for all $x \in [1, 3]$.

   (ii) the function $g: [1, 2] \to [1, 2]$ with $g(x) = 4x - x^2 - 2$ for all $x \in [1, 2]$.

3. Consider the graph Γ with vertices a, b, c, d, e and edges ab, bc, cd, de, ea, ac, ad. [Justify all your answers.]

   (a) Is the graph complete?

   (b) Is the graph regular?

   (c) Is the graph connected?

   (d) Does the graph have an Eulerian circuit?

   (e) Does the graph have a Hamiltonian circuit?

   (f) Give an example of a spanning tree for the graph, specifying the vertices and edges of the spanning tree.