Course 2BA1: Michaelmas Term 2003. Assignment II.

To be handed in by Friday 28th November, 2003. Please include both name and student number on any work handed in.

- 1. Prove that $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$ for all sets A, B and C.
- 2. For each of the following relations, determine whether or not that relation is reflexive, symmetric, transitive, anti-symmetric, an equivalence relation, and/or a partial order, giving appropriate reasons for your answers:—

(i) the relation P on the set \mathbb{R} of real numbers, where real numbers x and y satisfy xPy if and only if $y = xk^2$ for some integer k.

(ii) the relation Q on the set \mathbb{R} of real numbers, where real numbers x and y satisfy xQy if and only if $y^3 = x^3 - x + y$.

3. For each of the following functions, determine whether or not that function is injective and/or surjective, and whether or not it has a well-defined inverse, giving appropriate reasons for your answers:—

(i) the function $f: [-1,1] \rightarrow [-2,2]$ with $f(x) = x^3 + x$ for all $x \in [-1,1]$.

(ii) the function $g: (-1, 1) \to \mathbb{R}$ with $g(x) = \frac{1}{1 - x^2}$ for all $x \in (-1, 1)$.