Course 2BA1: Michaelmas Term 2002.

Assignment II.

To be handed in by Friday 22nd November, 2002.
Please include both name and student number on any work handed in.

1. Prove that $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ for all sets A, B and C.

2. For each of the following relations, determine whether or not that relation is reflexive, symmetric, transitive, anti-symmetric, an equivalence relation, and/or a partial order, giving appropriate reasons for your answers:—

   (i) the relation P on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xPy if and only if xy is odd.

   (ii) the relation Q on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xQy if and only if there exists an integer k such that $x = 2^k(y-1)+1$.

3. For each of the following functions, determine whether or not that function is injective and/or surjective, and whether or not it has a well-defined inverse, giving appropriate reasons for your answers:—

   (i) the function $f\colon [-1,1] \to [-2,2]$ with $f(x) = x^3 - x$ for all $x \in [-1,1]$.

   (ii) the function $g\colon [1,2] \to [0,6]$ with $g(x) = x^3 - x$ for all $x \in [1,3]$.

   (iii) the function $h\colon [0,1] \to [-2,2]$ with $h(x) = x^3 - x$ for all $x \in [0,1]$.