Course 2BA1: Michaelmas Term 2001.

Assignment II.

To be handed in by Friday 23rd November, 2001.
Please include both name and student number on any work handed in.

1. Prove that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ for all sets A, B and C.

2. For each of the following relations, determine whether or not that relation is reflexive, symmetric, transitive, anti-symmetric, an equivalence relation, and/or a partial order, giving appropriate reasons for your answers:—

   (i) the relation P on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xPy if and only if $x^2 \leq y^2$;

   (ii) the relation Q on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xQy if and only if $x - y = k^3$ for some integer k;

   (iii) the relation R on the set $\mathbb{N}$ of natural numbers, where natural numbers m and n satisfy mRn if and only if $m + 1$ divides $n + 1$ (i.e., if and only if $n + 1 = k(m + 1)$ for some integer k);

   (iv) the relation S on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xSy if and only if xy is even.

3. For each of the following functions, determine whether or not that function is injective and/or surjective, and whether or not it has a well-defined inverse, giving appropriate reasons for your answers:—

   (i) the function $f\colon \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ with $f(1) = 2$, $f(2) = 3$, $f(3) = 4$ and $f(4) = 5$ and $f(5) = 3$;

   (ii) the function $g\colon [4, 5] \to [9, 12]$ with $g(x) = 4 + 6x - x^2$ for all $x \in [4, 5]$;

   (iii) the function $h\colon [2, 4] \to [12, 13]$ with $h(x) = 4 + 6x - x^2$ for all $x \in [2, 4]$.