

Course 2BA1: Academic Year 2000–1.

Assignment II.

To be handed in by Friday 1st December, 2000.

Please include both name and student number on any work handed in.

- For each of the following relations on the set \mathbb{N} of natural numbers, determine whether or not that relation is reflexive, symmetric, transitive, anti-symmetric, an equivalence relation, and/or a partial order, giving appropriate reasons for your answers:—
 - the relation $|$ on the set \mathbb{N} of natural numbers, where natural numbers m and n satisfy $m|n$ if and only if m divides n ;
 - the relation P on the set \mathbb{N} of natural numbers, where natural numbers m and n satisfy mPn if and only if $n = 2^k m$ for some integer k (which may be positive, zero or negative);
 - the relation Q on the set \mathbb{N} of natural numbers, where natural numbers m and n satisfy mQn if and only if $m + n$ is divisible by 2;
 - the relation R on the set \mathbb{N} of natural numbers, where natural numbers m and n satisfy mRn if and only if $m + n$ is divisible by 3.
- For each of the following functions, determine whether or not that function is injective and/or surjective, and whether or not it has a well-defined inverse, giving appropriate reasons for your answers:—
 - the function $f: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ with $f(1) = 2$, $f(2) = 3$, $f(3) = 2$ and $f(4) = 4$;
 - the function $g: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ with $f(1) = 2$, $f(2) = 3$, $f(3) = 1$ and $f(4) = 4$;
 - the function $h: [1, 2] \rightarrow [0, \frac{1}{2}]$ with $h(x) = \frac{x-1}{x}$, where $[1, 2] = \{x \in \mathbb{R} : 1 \leq x \leq 2\}$ and $[0, \frac{1}{2}] = \{x \in \mathbb{R} : 0 \leq x \leq \frac{1}{2}\}$.
- If a complete graph has n vertices, how many edges does it have? (Justify your answer.)