Course 2BA1: Michaelmas Term 2008.

Assignment I.

To be handed in by Wednesday 12th November, 2008.
Please include both name and student number on any work handed in.

1. Let $x_1, x_2, x_3, \ldots$ be an infinite sequence with $x_1 = 1$, $x_2 = 3$ and $x_{n+2} = 4x_{n+1} - 3x_n$ for all positive integers n. Use the Method of Mathematical Induction to prove that $x_n = 3^{n-1}$ for all positive integers n.

2. Let A, B and C be sets. Prove that

$$A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A).$$

   (Here $B \setminus C$ denotes the set consisting of all elements of the set B that do not belong to the set C.)

3. Let R denote the relation on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xRy if and only if $x^2 - y^2$ is divisible by 7. Determine whether or not the relation R on $\mathbb{Z}$ is (i) reflexive, (ii) symmetric, (iii) anti-symmetric, (iv) transitive (v), an equivalence relation, (vi) a partial order. [Briefly justify your answers.]

4. Let Q denote the relation on the set $\mathbb{R}$ of real numbers, where real numbers x and y satisfy xQy if and only if $(x - y)^2 < 1$. Determine whether or not the relation R on $\mathbb{Z}$ is (i) reflexive, (ii) symmetric, (iii) anti-symmetric, (iv) transitive (v), an equivalence relation, (vi) a partial order. [Briefly justify your answers.]