Course 2BA1: Michaelmas Term 2007.

Assignment I.

To be handed in by Wednesday 14th November, 2007.
Please include both name and student number on any work handed in.

1. Use the Method of Mathematical Induction to prove that

$$\sum_{k=1}^{n} k^3 > \tfrac{1}{4}n^4$$

    for all positive integers n.

2. Let A, B and C be sets. Prove that

$$A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C).$$

    (Here $B \setminus C$ denotes the set consisting of all elements of the set B that do not belong to the set C.)

3. Let R and S denote the relations on the set $\mathbb{Z}$ of integers, where xRy if and only if $x - y$ is divisible by 5, and xSy if and only if $x + y$ is divisible by 5. Is the relation R an equivalence relation on $\mathbb{Z}$? Is the relation S an equivalence relation on $\mathbb{Z}$?

4. Let $f : [0, 3] \to [0, 9]$ be the function defined by

$$f(x) = 2x^3 - 9x^2 + 12x.$$

    Is the function f injective? Is it surjective? Is it invertible?