Course 2BA1: Michaelmas Term 2005.

Assignment I.

To be handed in by Wednesday 16th November, 2005.
Please include both name and student number on any work handed in.

1. Use the Method of Mathematical Induction to prove that

$$\sum_{k=1}^{n} \frac{1}{(k+2)(k+3)(k+4)} = \frac{1}{24} - \frac{1}{2(n+3)(n+4)}.$$

   for all positive integers n.

2. Let A, B and C be sets. Prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

3. Let $\sim$ denote the relation on the set $\mathbb{Z}$ of integers, where integers x and y satisfy $x \sim y$ if and only if xy is divisible by 3. Is the relation $\sim$ reflexive? Is it symmetric? Is it transitive? Is it an equivalence relation? Is it a partial order? [Justify your answers.]

4. Let R denote the relation on the set $\mathbb{N}$ of natural numbers, where natural numbers m and n satisfy mRn if and only if $n = 2^k m$ for some integer k (which may be positive, zero or negative). Is the relation R reflexive? Is it symmetric? Is it transitive? Is it an equivalence relation? Is it a partial order? [Justify your answers.]

5. Let $f\colon [0,3] \to [1,5]$ be the function defined by

$$f(x) = 4 + 2x - x^2.$$

   Is the function f injective? Is it surjective? Is it invertible?