Course 221: Hilary Term 2008.

Assignment II.

To be handed in by Wednesday, February 20, 2008.
Please include both name and student number on any work handed in.

1. Let X be a topological space. Prove that the union of any finite number of compact subsets of X is itself a compact subset of X.

2. Let X be a topological space, let Y be a Hausdorff space, let $f\colon X \to Y$ be a continuous function from X to Y, and let G be the subset of $X \times Y$ defined by

   $$G = \{(x, y) \in X \times Y : y = f(x)\}.$$

   Prove that G is a closed set in $X \times Y$ (where the topology on $X \times Y$ is the product topology.

   [Hint: don't panic straight away! A routine proof of this result makes direct use of the definition of a closed subset of a topological space, the definition of the product topology, the definition of a Hausdorff space, and the definition of a continuous function between topological spaces. An understanding of these definitions should be sufficient to enable you to come up with a proof of this result.]

[Remember: the above questions concern topological spaces. Topological spaces need not be metric spaces, and thus you cannot make reference to either distance functions or open balls in proofs of results concerning general topological spaces.]