
Assignment II.

To be handed in by Monday 19th February, 2007.
Please include both name and student number on any work handed in.

1. Evaluate
\[ \int_{-\infty}^{\infty} \frac{e^{isx}}{(x^2 + 1)(x^2 + 4)} \, dx \]
for all positive real numbers \( s \). [Hint: apply Cauchy’s Residue Theorem to the path integral taken around a closed path (or contour) that traverses the real axis from \(-R\) to \(R\), where \( R \) is a large positive real number, and then returns from \( R \) to \(-R\) in the anticlockwise direction around a semicircle of radius \( R \) about zero in the upper half plane.]

2. Evaluate
\[ \int_{0}^{+\infty} \frac{x^\alpha}{x(x^2 + 1)(x^2 + 4)} \, dx \]
for all real numbers \( \alpha \) satisfying \( 0 < \alpha < 1 \). [Hint: let \( R \) be a large positive real number, let \( e \) be a small positive real number, and apply Cauchy’s Residue Theorem to the path integral of
\[ \frac{z^\alpha}{z(z^2 + 1)(z^2 + 4)} \]
taken round a contour that traverses a straight line from \(-R + ie\) to \((1+i)e\), then traverses part of a circle of radius \( \sqrt{2e} \) centered on zero in a clockwise direction from \((1+i)e\) to \((1-i)e\), then traverses a straight line from \((1-i)e\) to \(-R - ie\), and finally traverses part of a circle of radius \( \sqrt{R^2 + e^2} \) centered on zero in an anti-clockwise direction from \(-R - ie\) to \(-R + ie\). Then let \( R \to +\infty \) and \( e \to 0 \). Here \( z^\alpha = \exp(\alpha \log z) \), where \( \log z \) denotes the principal branch of the logarithm function, defined throughout \( \mathbb{C} \setminus \{ t \in \mathbb{R} : t \leq 0 \} \).]