Course 214: Trinity Term 2008.

Assignment I.

To be handed in by Wednesday 30th April, 2008.
Please include both name and student number on any work handed in.

1. Determine the winding number of the closed curve $\gamma\colon [0,1] \to \mathbb{C}$ about zero, where

   $$\gamma(t) = 4\cos 14\pi t + \sin 22\pi t + (6\sin 14\pi t - 2\cos 22\pi t)i.$$

   [Briefly justify your answer.]

2. Determine which of the following functions are holomorphic:

   (i) The function $f_1\colon \mathbb{C} \to \mathbb{C}$ defined such that $f_1(z) = z^3 + 1$ for all $z \in \mathbb{C}$;

   (ii) The function $f_2\colon \mathbb{C} \setminus \{1\} \to \mathbb{C}$ defined such that $f_1(z) = \dfrac{1}{1-z}$ for all $z \in \mathbb{C} \setminus \{1\}$.

   (iii) The function $f_3\colon \mathbb{C} \to \mathbb{C}$ defined such that $f_3(z) = \dfrac{1}{|z|^2 + 1}$ for all $z \in \mathbb{C}$.

   [Briefly justify your answers.]

3. The functions sinh and cosh are defined by the formulae

   $$\sinh z = \frac{1}{2}(\exp z - \exp(-z)), \quad \cosh z = \frac{1}{2}(\exp z + \exp(-z)).$$

   Prove that

   $$\cos(z + iw) = \cos z \cosh w - i \sin z \sinh w$$

   for all complex numbers z and w.

4. What is the value of the path integral $\displaystyle\int_\gamma \frac{dz}{z-2}$ where $\gamma\colon [0,\pi] \to \mathbb{C}$ is the path with $\gamma(t) = \cos t + i \sin t$ for all $t \in [0,\pi]$? [Note: there are some methods for evaluating this integral, which require significantly less effort than is involved in a direct calculation using the formula that defines the value of the path integral.]