Course 214: Michaelmas Term 2006.

Assignment I.

To be handed in by Friday 1st December, 2006.
Please include both name and student number on any work handed in.

1. Determine which of the following subsets of the complex plane $\mathbb{C}$ are open in $\mathbb{C}$, and also which are closed in $\mathbb{C}$:—

   (i) $\{z \in \mathbb{C} : |z| < 3 \text{ and } \operatorname{Re} z > 1\}$;

   (ii) $\{z \in \mathbb{C} : |z| < 3 \text{ and } \operatorname{Re} z \geq 1\}$;

   (iii) $\{z \in \mathbb{C} : \operatorname{Re} z \leq 1 \text{ and } \operatorname{Im} z \leq 2\}$.

   [Briefly justify your answers.]

2. Determine the winding number of the closed curve $\gamma\colon [0,1] \to \mathbb{C}$ about zero, where

$$\gamma(t) = 5\cos 10\pi t + \sin 36\pi t + (4\sin 10\pi t - \cos 36\pi t)i.$$

   [Briefly justify your answer.]

3. Determine which of the following functions are holomorphic on their domains:

   (i) The function $f_1\colon \mathbb{C} \to \mathbb{C}$ defined such that $f_1(z) = \overline{z}$ for all $z \in \mathbb{C}$;

   (ii) The function $f_2\colon \mathbb{C} \setminus \{1, -1\} \to \mathbb{C}$ defined such that $f_1(z) = \dfrac{1}{1 - z^2}$ for all $z \in \mathbb{C} \setminus \{1, -1\}$.

   (iii) The function $f_3\colon \mathbb{C} \setminus \{0\} \to \mathbb{C}$ defined such that $f_3(z) = |z|^2$ for all $z \in \mathbb{C}$.

   [Briefly justify your answers.]