

1. Give a definition of the vector space.

2. Consider the vector space $V \simeq \mathbb{R}^n$.

Dual vector space V^* to V is defined as the space of all linear functions $f : V \rightarrow \mathbb{R}$.

- Prove that V^* is a vector space,
- Find dimension of V^* .

3. Give a definition of

- differential 1-form
- differential of a scalar field

4. Consider the following differential forms:

$$dx + dy, \quad xdy + dx, \quad \frac{x}{y^2}dy - \frac{dx}{y} \quad (1)$$

- Which of these forms are differentials of a scalar field?
- Find the corresponding scalar field (if it exists)
- Integrate all three forms along a) a straight line from $\{x, y\} = \{0, 0\}$ to $\{x, y\} = \{1, 1\}$ b) a path consisting of two straight lines that connect points $\{x, y\} = \{0, 0\}$, $\{x, y\} = \{1, 0\}$, $\{x, y\} = \{1, 1\}$.