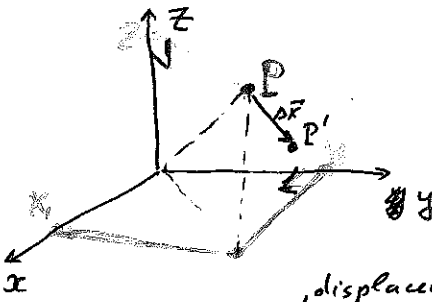
| Goldstein |

In physics you need means to describe ~~positi~~ objects



$(t; x, y, z)$

$(t'; x', y', z')$

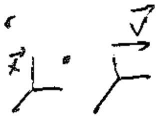
$$\frac{\Delta \vec{x}}{\Delta t} = \vec{v}$$

displacement ($\Delta\vec{x}$); $\vec{v}$ ← velocity of the particle; Δt ← time to make displacement

Inertial frame: a coordinate system in which ~~part~~ $\vec{v}$ remains const in time, if no forces acting on the particle.

- All inertial frames move with constant speeds one with respect to another

$$\vec{x}' = \vec{x} - \vec{v}t$$



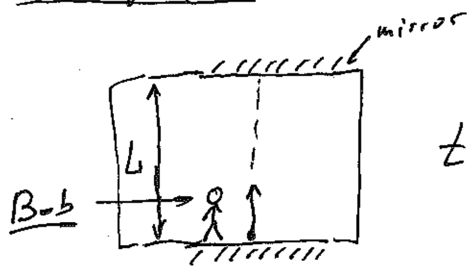
- Galilean ~~relati~~ principle of relativity: Physical laws are ~~all~~ the same in any inertial frame

- Einstein's principle of relativity: Physical laws are the same in any ~~inter~~ inertial frame.

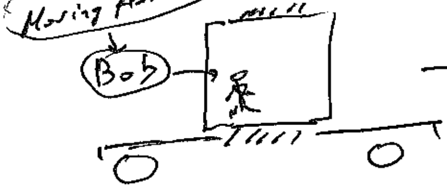
  - speed of light is constant.] ← physical law! here is the difference.

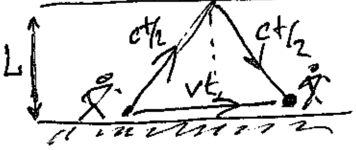
$$\boxed{c = 299792458\ \text{m/s}} \simeq 3\cdot 10^8\ \text{m/s}$$

Measuring of time



$$t = \frac{2L}{c}; \quad \frac{t}{2} = \frac{L}{c}$$





Frame of external observer (Alice)

$$\left(v\frac{t}{2}\right)^2 + L^2 = \left(c\frac{t}{2}\right)^2$$

$$\frac{t}{2} = \frac{L}{\sqrt{c^2 - v^2}} = \frac{1}{\sqrt{1 - v^2/c^2}}\frac{L}{c} = \frac{L}{c}\gamma$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}; \quad v/c = \beta$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

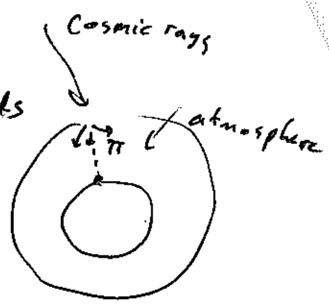
Conclusion:

$$\frac{t_B}{2} = \frac{L}{c}$$

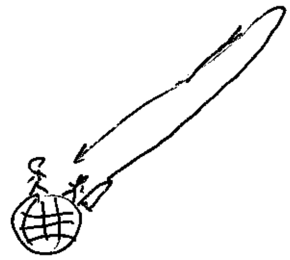
$$\frac{t_A}{2} = \gamma\cdot\frac{L}{c}, \quad \gamma > 1$$

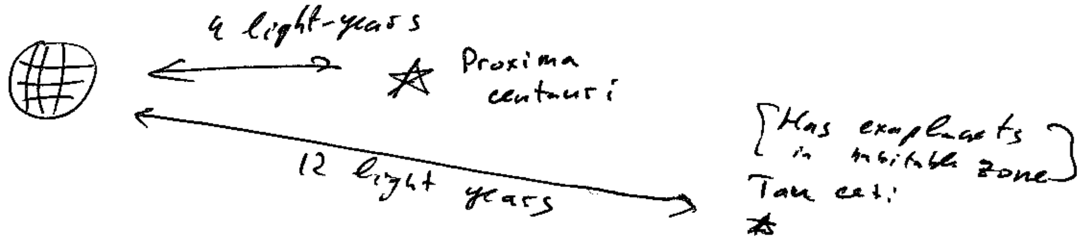
Examples ~~from real life~~ ②

• π-meson — decays in ~ ~~30 nano-seconds~~ $2.6 \cdot 10^{-8}$ s



• Twin paradox:





$$t_{Earth} = 12 \text{ light years } /\beta \; ; \quad \beta = v/c = 0.99$$

$$\cancel{t_{rocket} = \frac{12 \text{ light years}/\beta}{\sqrt{1-\beta^2}} = \frac{12 \text{ l.y.}}{\beta}\left(1 + \tfrac{1}{2}\right.}$$

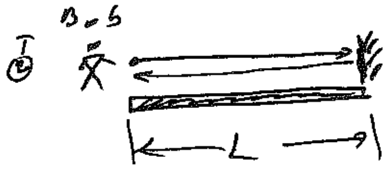
$$t_{rocket} = t_{Earth}\sqrt{1-\beta^2} \;\; \cancel{\approx t_{Earth}\sqrt{0.2} = 0.4}\; t_{Earth}.$$

β	$\sqrt{1-\beta^2}$	time [years] to τ-ceti / Earth	rocket
0.1	0.995	120	119
0.5	0.87	24	21
0.9	0.44	13	6
0.99	0.14	12	2
0.999	0.04	12	0.5

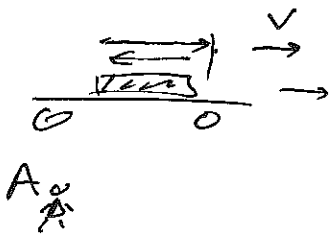
current ~~rocket speed~~ speed of voyager ~~622 km/h~~ $1.7 \cdot 10^4$ m/s (62000 $\frac{km}{h}$)

$$\beta \simeq 0.5 \cdot 10^{-4}$$

Measuring of distance



$$L_B = \frac{1}{2} c \cdot t_B$$



$$t_{\rightarrow} = \frac{L}{c-V}$$

$$t_{\leftarrow} = \frac{L}{c+V} \qquad (1)$$

$$t_{\rightarrow} + t_{\leftarrow} = t_A$$

$$t_{\rightarrow} / t_{\leftarrow} = \frac{c+V}{c-V}$$

$$\left(\frac{c+V}{c-V} + 1\right) t_{\leftarrow} = t_A;$$

$$t_{\leftarrow} = t_A \cdot \frac{c-V}{2c} \qquad (2)$$

$$\begin{matrix}(1)\to \\ (2)\end{matrix} \quad L = \frac{c^2 - V^2}{2c} t_A = \left(1 - \frac{V^2}{c^2}\right) \frac{1}{2} c \cdot t_A =$$

$$L_A = \frac{1}{\gamma^2} \frac{1}{2} c t_A = \frac{1}{\gamma^2} \left(\frac{1}{2} c \cdot t_B\right) \cdot \gamma = \frac{L_B}{\gamma} \qquad \underline{\gamma > 1}$$

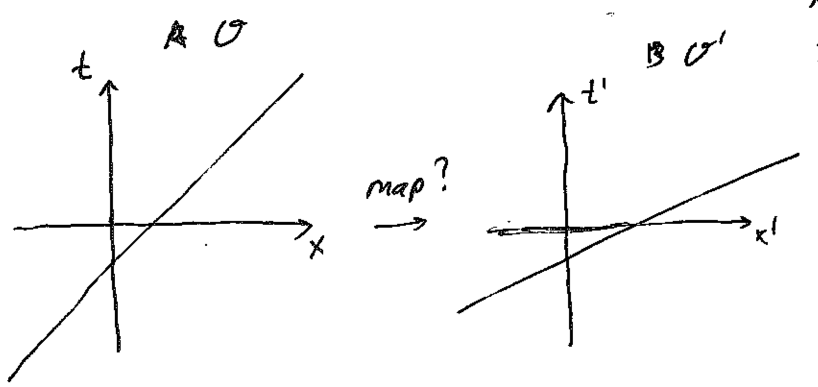
$$\boxed{L_A = \frac{L_B}{\gamma}}$$ ← Lorentz contraction.

GPS

$$\boxed{t_A = \gamma t_B}$$

$x' = x - vt$

~~$t' = t$~~



we want that lines go to lines [principle that $\vec{v} = \text{const}$ if no forces applied]

<u>Hence it should be a linear transformation</u> $\begin{pmatrix} t' \\ x' \end{pmatrix} = \hat{A} \cdot \begin{pmatrix} t \\ x \end{pmatrix} + \dots$

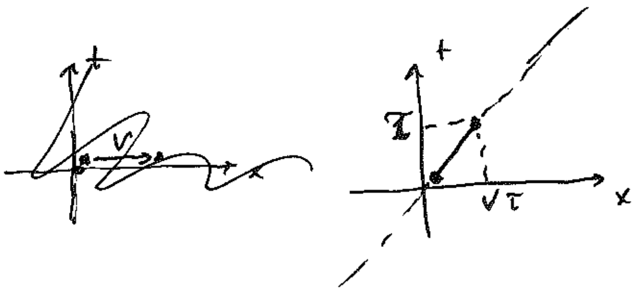
· Let us choose $\begin{matrix} t=0 \\ x=0 \end{matrix} \leftrightarrow \begin{matrix} t'=0 \\ x'=0 \end{matrix}$ ← just the question is where our origin

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \hat{A} \begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \alpha t + \beta x \\ \gamma t + \delta x \end{pmatrix} \qquad \hat{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

Consider first $x' = \gamma t + \delta x$

moving with velocity v means that

$$x' = C(-vt + x)$$



What is C?

put $t = 0$ $\quad x' = \delta x \qquad \boxed{C = \delta \text{ ?}}$

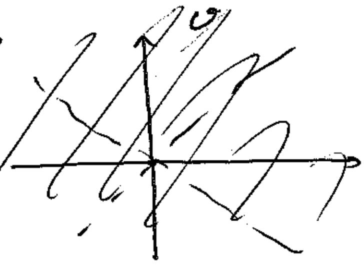
distance to origin   distance to origin

~~$t' = At + Bx$~~

~~If O'~~ moves ~~with v with~~ respect

~~Consider~~ $\cancel{t' = At + Bx}$

~~put~~ $\cancel{x = 0}$



~~Now let us prove the following statement~~

$\cdot \quad x' = \gamma(x - vt) \qquad$ can. $\quad O \to O'$

Consider opposite transform $\quad O' \to O$

$\cdot \quad x = \gamma(x' + vt')$

$\to \quad \cancel{t' = \gamma(t - v}$

$$t' = \frac{1}{v}\left(\frac{x}{\gamma} - x'\right) = \frac{1}{v}\left(\frac{x}{\gamma} - \gamma x + \gamma v t\right) =$$

$$= \gamma\left(t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right) =$$

$$= \gamma\left(t + \frac{x}{v}\left(\left(1 - \frac{v^2}{c^2}\right) - 1\right)\right) =$$

$$= \gamma\left(t - \frac{vx}{c^2}\right)$$

Lorentz transformation:

$$\boxed{\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) \end{cases}} \qquad \begin{cases} x' = \gamma(x - \beta(ct)) \\ ct' = \gamma(ct - \beta x) \end{cases}$$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\vec{x}' = \gamma(\vec{x} - \vec{\beta}(ct))$$

$$ct' = \gamma(ct - \vec{\beta}\cdot\vec{x})$$

~~Interval~~

Analogy: 3-dimensional space

Consider all possible $\overbrace{}^{\text{nondegenerate}}$ linear transformations

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \underset{\text{3×3 matrix}}{\hat{G}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \det G \neq 0$$

Consider a subspace of transformations O that preserve distances:

$$\vec{V}^2 = \sum_{i=1}^{3} v_i^2 = \sum_{i,j} \delta_{ij} V_i V_j$$

$$\vec{V} \cdot \vec{W} = \sum \delta_{ij} V_i W_j$$

$$V'_{i'} = O_{i'}{}^{i} V_i \qquad \delta_{i'j'} V'_{i'} W'_{j'} = \delta_{ij} V_i W_j$$

$$W'_{j'} = O_{j'}{}^{j} W_j \qquad (\delta_{i'j'} O_{i'}{}^{i} O_{j'}{}^{j}) V_i W_j = \delta_{ij} V_i W_j$$

$$\boxed{\delta_{i'j'} O_{i'}{}^{i} O_{j'}{}^{j} = \delta_{ij}}$$

Orthogonal transformations

$O(3)$

[All possible rotations $\leftarrow \det O = 1$

or reflections + rotations $\leftarrow \det O = -1$]

$$O O^T = \mathbb{1}$$

$$\Downarrow$$

$$1 = \det(O O^T) = [\det(O)]^2 = 1$$

$$\det O = \pm 1$$

Minkowski space — $1+3$ 4 dimensional vector space with metric

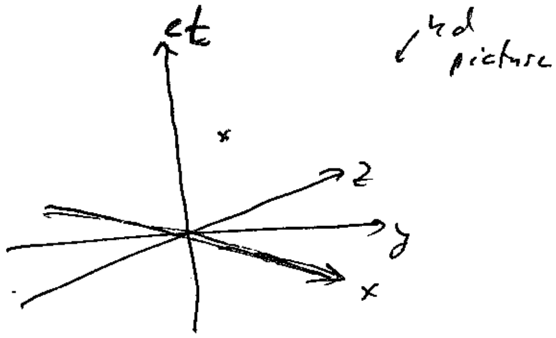
[space-time]

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

"Length"

Points in this space are called space-time events

$$\bar{X} = \{ct, x, y, z\}$$



1+1 dimension space

"Length" between events $\{ct_2, x_2, y_2, z_2\}$ and $\{ct_1, x_1, y_1, z_1\}$

is defined by

$$\eta_{\mu\nu}(\Delta X)^\mu (\Delta X)^\nu = \boxed{c^2(\Delta t)^2 - (\Delta \vec{x})^2} = (\Delta s)^2$$

← interval

Δs^2 can be >0, <0 so it is called

Consider all possible transformations that preserve interval

$$\boxed{O(1,3) \quad \det L = \pm 1}$$

$SO(1,3)$ $\det L = 1$

$$\boxed{\eta_{\mu'\nu'} L_\mu{}^{\mu'} L_\nu{}^{\nu'} = \eta_{\mu\nu}}$$

or $L \eta L^T = \eta$

$$\det(L\eta L^T) = \det \eta = 1$$

$$\det L \cdot \det \eta \cdot \det L^T = \det \eta$$

$$\boxed{\det L = \pm 1}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

← normal 3d rotations