FIRST ANSWER IS THE CORRECT ONE

All questions

differential geometry and related

dual vector space

Q:Find the wrong statement.

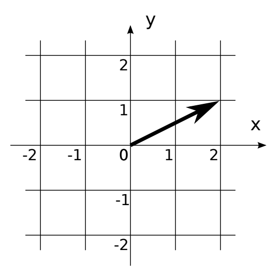
(A) We need a metric to define the dual vector space
(B) Dual space of V is the space of linear functionals on V
(C) Dimensions of dual space and V are equal assuming V is finite-dimensional
(D) Metric establishes a natural isomorphism between V and its dual
(E) $(V^*)^*$ is isomorphic to V assuming V is finite-dimensional

Q:Find the wrong statement.

(A) For each vector from V, there is a canonically dual vector in V^*
(B) Dual space of V is the space of linear functionals on V
(C) Dimensions of dual space and V are equal assuming V is finite-dimensional
(D) For each basis in V, there is a canonically dual basis in V^*.
(E) $(V^*)^*$ coincides with V assuming V is finite-dimensional

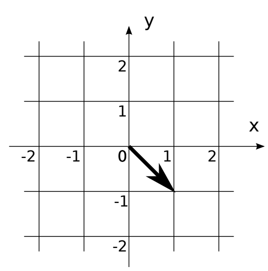
value of differential form

Q:Find the value of $dx + dy$ on the following vector:

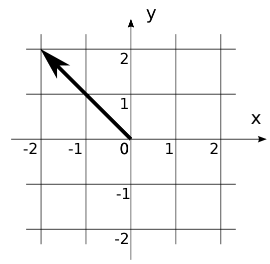


Answer:3

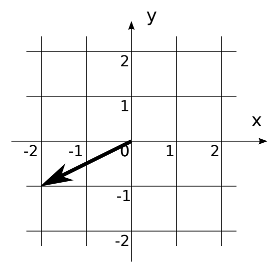
Q:Find the value of $2dx + dy$ on the following vector:



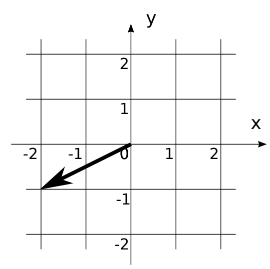
Answer:1

Q:Find the value of $2dx - dy$ on the following vector: 

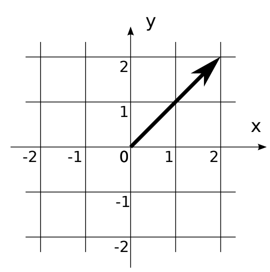
Answer:-6

Q:Find the value of $dx + dy$ on the following vector: 

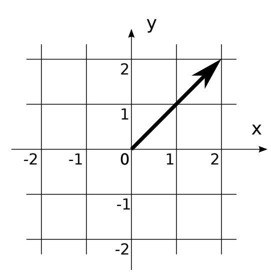
Answer:-3

Q:Find the value of $dx - 4dy$ on the following vector: 

Answer:2

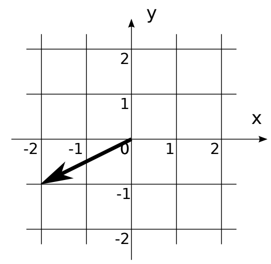
Q:Find the value of dx on the following vector: 

Answer:2

Q:Find the value of $dx - dy$ on the following vector: 

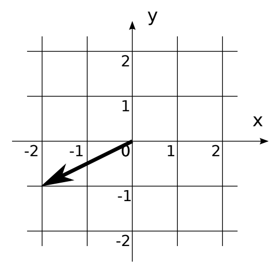
Answer:0

Q:Find the value of $dx + dy$ on the following vector:



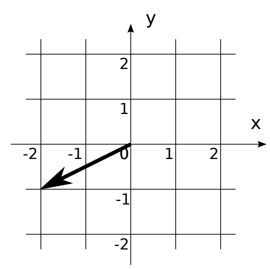
Answer:-3

Q:Find the value of $-dx + dy$ on the following vector:



Answer:1

Q:Find the value of $3dx - 2dy$ on the following vector:



Answer:-4

differential form in different frames

$x\,dy - y\,dx$ becomes in the polar coordinates

(A) $r^2\,d\phi$ (B) $d\phi$
(C) $-r\,d\phi$ (D) $\cos(\phi)\,dr$
(E) $\phi\,dr - r\,d\phi$

$x\,dy + y\,dx$ becomes in the polar coordinates

(A) $\cos(2\phi)r^2\,d\phi$ (B) $r^2\,d\phi$
(C) $d\phi$ (D) $-r\,d\phi$
(E) $\phi\,dr - r\,d\phi$

dx becomes in the complex coordinates

(A) $\frac{1}{2}\,(dz + d\bar{z})$ (B) $\frac{1}{2}\,(dz - d\bar{z})$
(C) $2\,(dz + d\bar{z})$ (D) dz
(E) $\sqrt{2}\,(dz - d\bar{z})$

$x\,dx + y\,dy$ becomes in the polar coordinates

(A) $\frac{1}{2}r\,dr$ (B) dr
(C) $d\phi$ (D) 0
(E) $2r^2\sin\phi\cos\phi\,d\phi$

Q:Rewrite rdr in the $\{x, y\}$ coordinates

Answer:$x\,dx + y\,dy$
Q:Rewrite $r^2 d\phi$ in the $\{x, y\}$ coordinates
Answer:$x\,dy - y\,dx$

Integration of differential form

Find $\int_\gamma dx + y\,dy$, where γ connects points $\{0, 0\}$ and $\{1, 1\}$

(A) 3/2 (B) 0
(C) 1/2 (D) 1
(E) 2 (F) 5/2

Find $\int_\gamma xdx - y\,dy$, where γ is a straight line connecting points $\{0, 0\}$ and $\{1, 1\}$

(A) 0 (B) 3/2
(C) 1/2 (D) 1
(E) 2 (F) 5/2

Find $\int_\gamma xdx +\ dy$, where γ connects points $\{-1, 0\}$ and $\{1, 1\}$

(A) 1 (B) 0
(C) 1/2 (D) -3/2
(E) 2 (F) 3/2

Find $\int_\gamma x\,dy - y\,dx$, where γ is a unit circle, integration is in a clock-wise direction

(A) -2π (B) π
(C) 0 (D) $+2\pi$
(E) 1 (F) -1

Find $\int_\gamma x\,dy - y\,dx$, where γ is a unit circle, integration is in a counter-clock-wise direction

(A) $+2\pi$ (B) π
(C) 0 (D) -2π
(E) 1 (F) -1

Find $\int_\gamma x\,dy + y\,dx$, where γ is a unit circle, integration is in a clock-wise direction

(A) 0 (B) π
(C) 2π (D) -2π
(E) 1 (F) -1

Scalar product

Q:For a vector space with metric $g_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, find the scalar product between vectors $(1, 0, 0)$ and $(0, 1, 0)$.
Answer:-1
Q:For a vector space with metric $g_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, find the scalar product between vectors $(1, 0, 0)$ and $(0, 0, 1)$.
Answer:0
Q:For a vector space with metric $g_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, find the scalar product between vectors $(0, 1, 0)$ and $(0, 0, 1)$.
Answer:-1
Q:For a vector space with metric $g_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, find the norm of the vector $(1, 0, 0)$.
Answer:$\sqrt{2}$
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Answer:$\sqrt{2}$
Q:For a vector space with metric $g_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, find the norm of the vector $(0, 0, 1)$.

Answer:$\sqrt{2}$

Legendre transform

Legendre transform of $2x^4$ is

(A) $\frac{3}{8}\,p^{4/3}$
(B) $\frac{1}{2}p^4$
(C) $p\,x - 2x^4$
(D) 0
(E) ill-defined
(F) $6x^4$

Legendre transform of $\sqrt{1+p^2}$ for $p>0$ is

(A) $-\sqrt{1-x^2}$
(B) $x\,p - \sqrt{1+p^2}$
(C) $\sqrt{1+x^2}$
(D) 0
(E) ill-defined

Legendre transform of $\cos x$ is a well-defined operation for

(A) $\frac{\pi}{2} < x < \frac{3\pi}{2}$
(B) $\frac{3\pi}{2} < x < \frac{5\pi}{2}$
(C) $\pi < x < 2\pi$
(D) $0 < x < \pi$
(E) any x

Legendre transform of $\sin x$ is a well-defined operation for

(A) $\pi < x < 2\pi$
(B) $0 < x < \pi$
(C) $\frac{\pi}{2} < x < \frac{3\pi}{2}$
(D) $\frac{3\pi}{2} < x < \frac{5\pi}{2}$
(E) any x

Lagrangians and Hamiltonians

Lagrangians in different coordinates

Lagrangian of a free particle of mass m in two dimensions in polar coordinates is

(A) $\frac{m}{2}\left(\dot r^2 + r^2\dot\varphi^2\right)$
(B) $\frac{m}{2}\left(\dot r^2 - r^2\dot\varphi^2\right)$
(C) $\frac{m}{2}\left(\dot r^2 + r^2\sin(2\phi)\dot\varphi^2\right)$
(D) $\frac{mr^2}{2}\dot\varphi^2$
(E) none of proposed

Lagrangian of a free particle of mass m in two dimensions which is constrained to move on a circle of radius r is

(A) $\frac{mr^2}{2}\dot\varphi^2$
(B) $\frac{m}{2}\left(\dot r^2 + r^2\dot\varphi^2\right)$
(C) $\frac{m}{2}\left(\dot r^2 - r^2\dot\varphi^2\right)$
(D) $\frac{m}{2}\left(\dot r^2 + r^2\sin(2\phi)\dot\varphi^2\right)$
(E) $\frac{mr^2}{2}\sin(2\phi)\dot\varphi^2$
(F) $\frac{m}{2}\dot\varphi^2$

Lagrangian of a free particle of mass m in two dimensions is written in complex coordinates $z = x + i\,y$, $\bar z = x - i\,y$ as

(A) $\frac{m}{2}\,\dot z\,\dot{\bar z}$
(B) $\frac{m}{2}\left(\dot z^2 + \dot{\bar z}^2\right)$
(C) $\frac{m}{2}\left(\dot z^2 - \dot{\bar z}^2\right)$
(D) $\frac{m}{2}\left(\dot z^2 + i\dot{\bar z}^2\right)$
(E) $m(\dot z + \dot{\bar z})^2$
(F) none of proposed

Lagrangian of a free particle of mass m in two dimensions is written in complex coordinates $z = x + i\,y$, $\bar z = x - i\,y$ as

(A) none of proposed
(B) $i\,\frac{m}{2}\,\dot z\,\dot{\bar z}$
(C) $\frac{m}{2}\left(\dot z^2 + \dot{\bar z}^2\right)$
(D) $\frac{m}{2}\left(\dot z^2 - \dot{\bar z}^2\right)$
(E) $\frac{m}{2}\left(\dot z^2 + i\dot{\bar z}^2\right)$
(F) $m(\dot z + \dot{\bar z})^2$

Lagrangian of a free particle of mass m in three dimensions in spherical coordinates is

(A) $\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\dot\varphi^2\right)$
(B) $\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + r^2\dot\varphi^2\right)$
(C) $\frac{m}{2r^2}\left(\dot r^2 + \dot\theta^2 + \dot\varphi^2\right)$
(D) $\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + r^2\sin\theta\dot\varphi^2\right)$

(E) $\frac{m}{2}\left(\dot{r}^2+\dot{\theta}^2+\dot{\varphi}^2\right)$ (F) none of proposed

Lagrangian of a free particle of mass m in three dimensions in spherical coordinates is

(A) none of the proposed
(B) $\frac{m}{2}\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin(2\theta)\dot{\varphi}^2\right)$
(C) $\frac{m}{2}\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\dot{\varphi}^2\right)$
(D) $\frac{m}{2r^2}\left(\dot{r}^2+\dot{\theta}^2+\dot{\varphi}^2\right)$
(E) $\frac{m}{2}\left(\dot{r}^2+r^2\dot{\theta}^2+\sin^2\theta\dot{\varphi}^2\right)$
(F) $\frac{m}{2}\left(\dot{r}^2+\dot{\theta}^2+\dot{\varphi}^2\right)$

Hamiltonians from lagrangians

Q:Lagrangian is given by $L=\dot{x}^2$. Write down the Hamiltonian (as a function of p)
Answer:$p^2/4$
Q:Lagrangian is given by $L=\frac{1}{2}\dot{q}^i\,M_{ij}\dot{q}^j$, summation over repeated index is assumed. Without loss of generality, we can assume that

(A) M is symmetric
(B) M is non-degenerate
(C) M is degenerate
(D) M is anti-symmetric
(E) none assumption about M can be made
(F) $\mathrm{tr}M=1$

Lagrangian is given by $L=\frac{1}{2}\dot{q}_i\,M_{ij}\dot{q}_j$, summation over repeated index is assumed. For M being a symmetric non-degenerate matrix that depends on q, the Hamiltonian is

(A) $H=\frac{1}{2}p_i\left(M^{-1}\right)_{ij}p_j$
(B) $H=\frac{1}{2}p_i\,M_{ij}\,p_j$
(C) $H=\frac{1}{2(\det M)^2}p_i\,M_{ij}\,p_j$
(D) $H=\frac{\det M}{2}p_i\left(M^{-1}\right)_{ij}p_j$
(E) none of the proposed

Lagrangian is given by $L=-m\sqrt{1-\dot{x}^2}$. The Hamiltonian is

(A) $\sqrt{m^2+p^2}$
(B) $m\sqrt{1+p^2}$
(C) $m\,c^2$
(D) $\sqrt{1+(p/m)^2}$
(E) $-\sqrt{1+(p/m)^2}$

Lagrangian is given by $L=\frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right)+B(x\,\dot{y}-y\,\dot{x})$. The Hamiltonian is

(A) $\frac{1}{2m}\left(p_x^2+p_y^2\right)-\frac{B}{m}\left(x\,p_y-y\,p_x\right)+\frac{B^2}{2m}\left(x^2+y^2\right)$
(B) $\frac{1}{2m}\left(p_x^2+p_y^2\right)+\frac{B}{m}\left(x\,p_y-y\,p_x\right)+\frac{B^2}{2}\left(x^2+y^2\right)$
(C) $\frac{1}{2m}\left(p_x^2+p_y^2\right)+\frac{2B}{m}\left(x\,p_y-y\,p_x\right)$
(D) $\frac{1}{2m}\left(p_x^2+p_y^2\right)-\frac{B}{m}\left(x\,p_y+y\,p_x\right)+\frac{B^2}{2m}\left(x^2+y^2\right)$

Poisson brackets I

Q:Compute the Poisson bracket $\{x^2,p^3\}$
Answer:$6\,p^2\,x$
Q:Compute the Poisson bracket $\{x^3+p,p^2\}$
Answer:$6\,p\,x^2$
Q:Compute the Poisson bracket $\{x^2+y^2,x\,p_y-y\,p_x\}$
Answer:0
Q:What is the value of the Poisson bracket $\{L_x,L_y\}$ (give the answer in terms of L_x,L_y,L_z)?
Answer:L_z
Q:What is the value of the Poisson bracket $\{L_x,L_z\}$ (give the answer in terms of L_x,L_y,L_z)?
Answer:$-L_y$

Q:What is the value of the Poisson bracket $\{L_z, L_y\}$ (give the answer in terms of L_x, L_y, L_z)?
Answer:$-L_x$
Q:What is the value of the Poisson bracket $\{L_x, L_x^2 + L_y^2 + L_z^2\}$?
Answer:0

Poisson brackets II
Q:Let $f(x,p,t)$ be a function of x, p, t. The equation $\frac{df}{dt} - \frac{\partial f}{\partial t} + \{H, f\} = 0$ tells us

(A) nothing, it is just true for any f
(B) that $\frac{\partial f}{\partial t} = 0$
(C) that f is a conserved quantity
(D) that $\{H, f\} = 0$
(E) this equation can only hold if $f \equiv 0$

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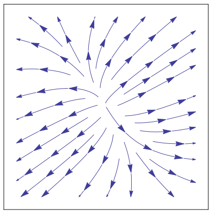
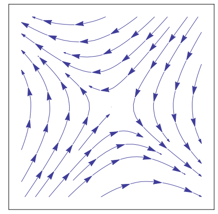
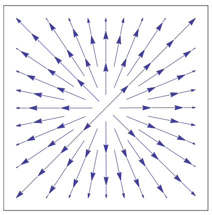
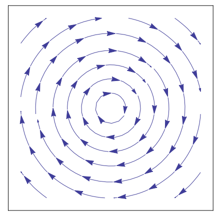
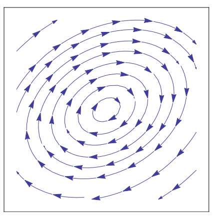
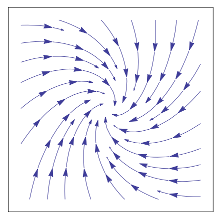
(B) nothing, it is just true for any f
(C) that $\frac{\partial f}{\partial t} = 0$
(D) that $\{H, f\} = 0$
(E) this equation can only hold if $f \equiv 0$

Q:Let for some functions $A(x,p,t)$ and $B(x,p,t)$ one has $\{A, H\} = 0$ and $\{B, H\} = 0$. What we can say about the quantity $C = \{A, B\}$?

(A) $\{C, H\} = 0$
(B) $C = 0$
(C) C is proportional to H
(D) $\frac{\partial C}{\partial t} = 0$
(E) $\frac{dC}{dt} = 0$

Phase portraits

PP from differential equation
The phase portrait of the system of equations $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$ with $A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ is

(A) 
(B) 
(C) 
(D) 
(E) 
(F) 

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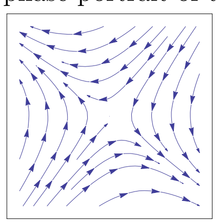
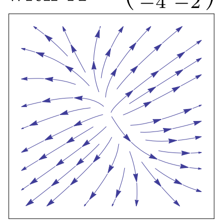
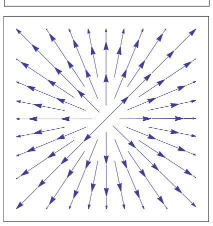
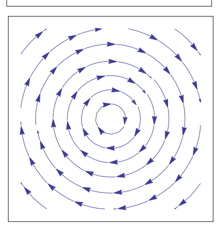
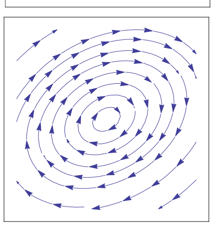
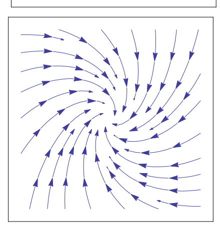
(A) 
(B) 
(C) 
(D) 
(E) 
(F) 

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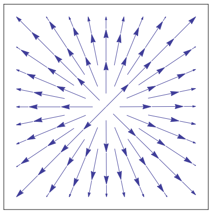
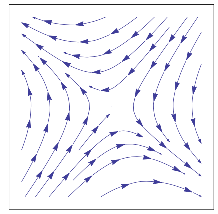
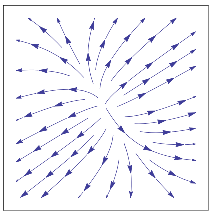
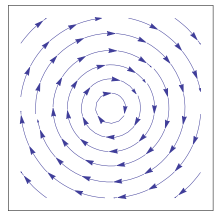
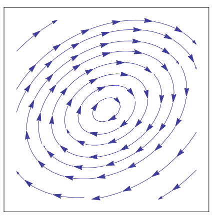
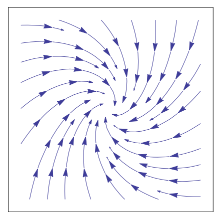
(A) 

(B) 

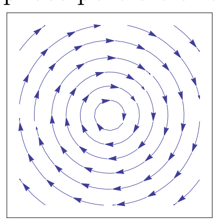
(C) 

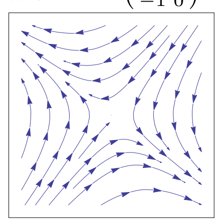
(D) 

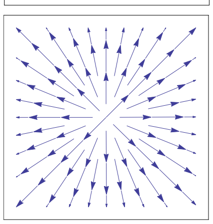
(E) 

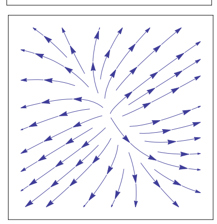
(F) 

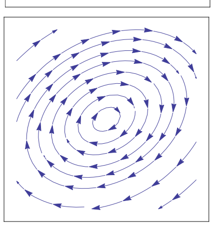
The phase portrait of the system of equations $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = A\begin{pmatrix}x\\ y\end{pmatrix}$ with $A = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ is

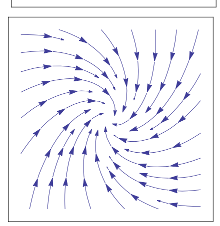
(A) 

(B) 

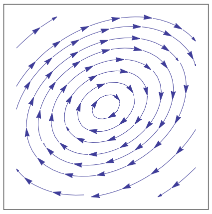
(C) 

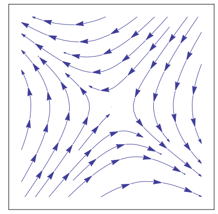
(D) 

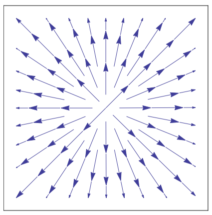
(E) 

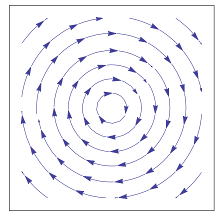
(F) 

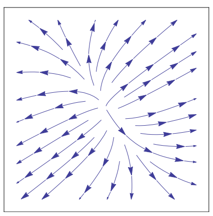
The phase portrait of the system of equations $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = A\begin{pmatrix}x\\ y\end{pmatrix}$ with $A = \begin{pmatrix}-1 & 4\\ -3 & 1\end{pmatrix}$ is

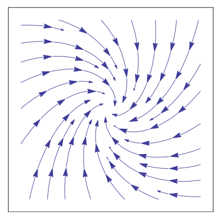
(A) 

(B) 

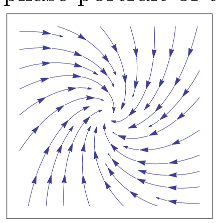
(C) 

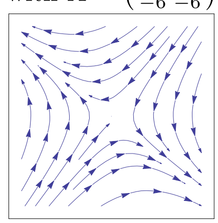
(D) 

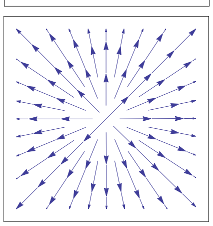
(E) 

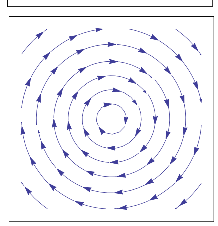
(F) 

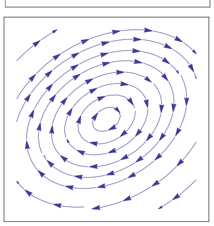
The phase portrait of the system of equations $\begin{pmatrix}\dot x\\ \dot y\end{pmatrix} = A\begin{pmatrix}x\\ y\end{pmatrix}$ with $A = \begin{pmatrix}-7 & 4\\ -6 & -6\end{pmatrix}$ is

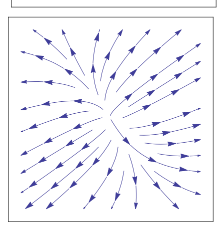
(A) 

(B) 

(C) 

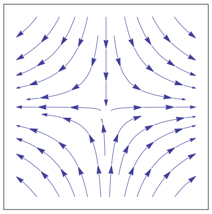
(D) 

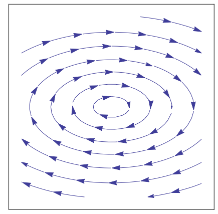
(E) 

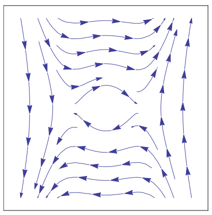
(F) 

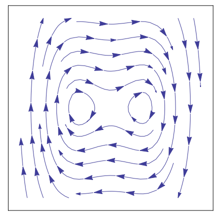
PP from hamiltonians

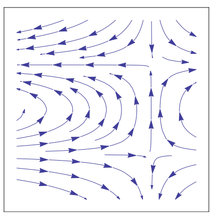
The phase portrait of the vector flow generated by the Hamiltonian $\mathcal{H} = \frac{1}{2}p\,x$ is

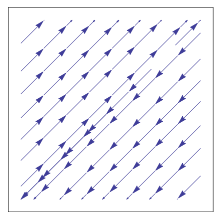
(A) 

(B) 

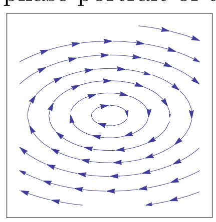
(C) 

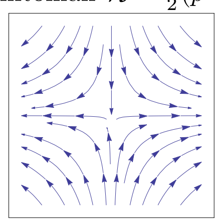
(D) 

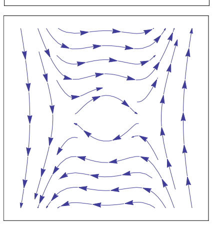
(E) 

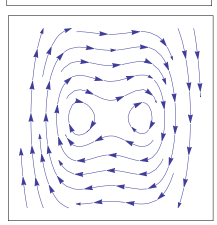
(F) 

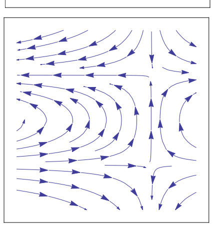
The phase portrait of the vector flow generated by the Hamiltonian $\mathcal{H} = \frac{1}{2}(p^2 + \frac{1}{3}x^2)$ is

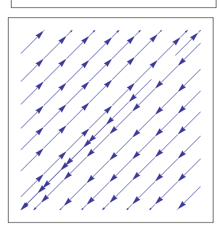
(A) 

(B) 

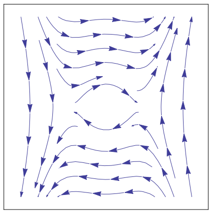
(C) 

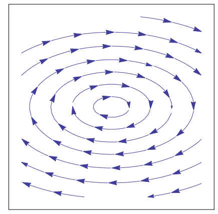
(D) 

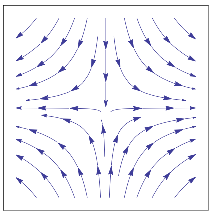
(E) 

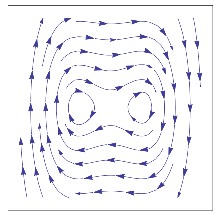
(F) 

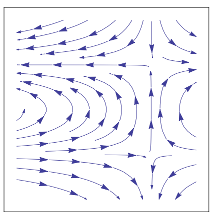
The phase portrait of the vector flow generated by the Hamiltonian $\mathcal{H} = \frac{1}{2}(p^2 + x^2 - x^4)$ is

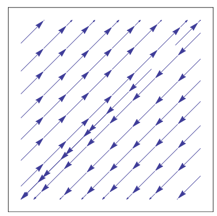

(A)


(B)

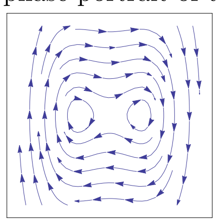

(C)

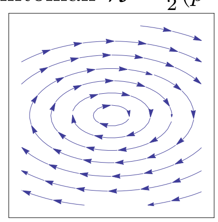

(D)

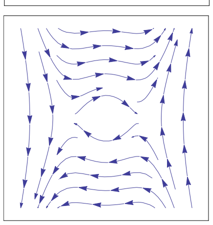

(E)

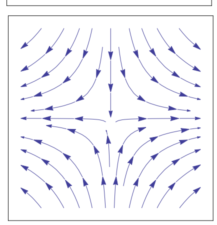

(F)

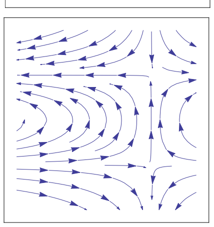
The phase portrait of the vector flow generated by the Hamiltonian $\mathcal{H} = \frac{1}{2}(p^2 - x^2 + x^4)$ is

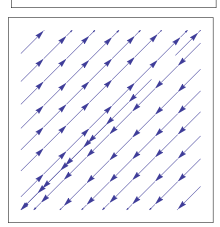

(A)


(B)

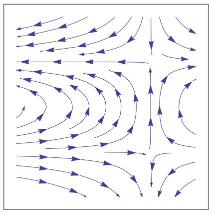

(C)

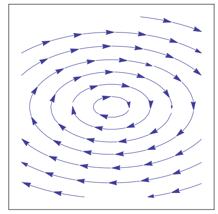

(D)

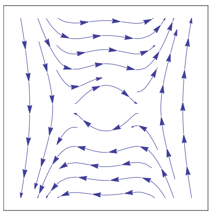

(E)

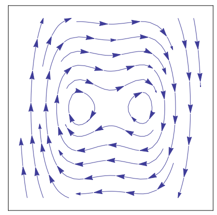

(F)

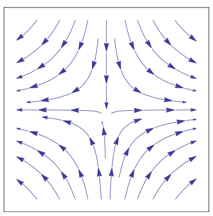
The phase portrait of the vector flow generated by the Hamiltonian $\mathcal{H} = \frac{1}{2}(p^2 - 1)(x - 1)$ is

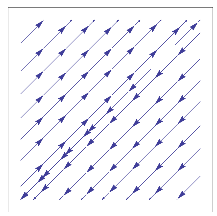
(A) 

(B) 

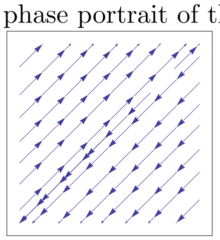
(C) 

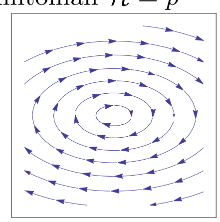
(D) 

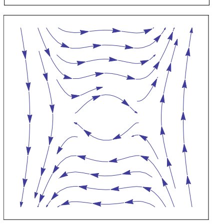
(E) 

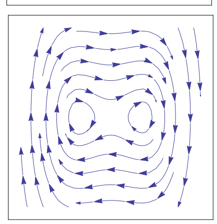
(F) 

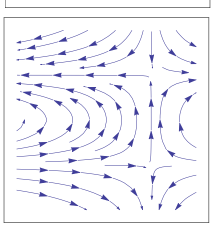
The phase portrait of the vector flow generated by the Hamiltonian $\mathcal{H} = p^2 - 2p\,x + x^2$ is

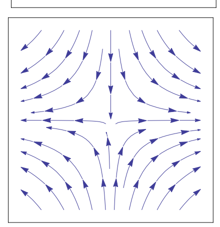
(A) 

(B) 

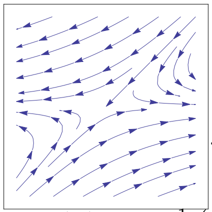
(C) 

(D) 

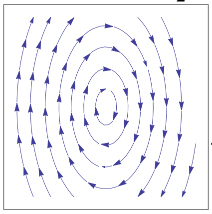
(E) 

(F) 

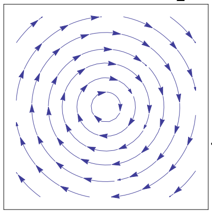
PP to hamiltonians

The phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

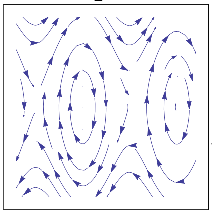
(A) $\mathcal{H} = p(x - p)$
(B) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$
(D) $\mathcal{H} = p(x + p)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$
(F) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$

The phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

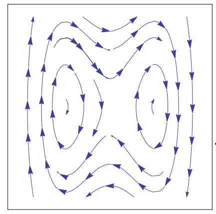
(A) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$
(B) $\mathcal{H} = p(x - p)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$
(D) $\mathcal{H} = \frac{1}{2}p^2 + \sin(3x)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$
(F) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$

The phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

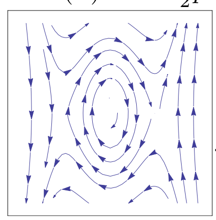
(A) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$
(B) $\mathcal{H} = p(x - p)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$
(D) $\mathcal{H} = \frac{1}{2}p^2 + \sin(3x)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$
(F) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$

A part of the phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

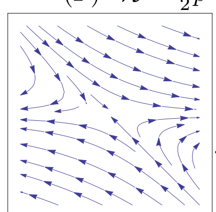
(A) $\mathcal{H} = \frac{1}{2}p^2 + \sin(3x)$
(B) $\mathcal{H} = p(x - p)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$
(D) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$
(F) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$

The phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

(A) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$ (B) $\mathcal{H} = p(x - p)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$ (D) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + \sin(3x)$ (F) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$

The phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

(A) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$ (B) $\mathcal{H} = p(x - p)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$ (D) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + \sin(3x)$ (F) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$

The phase portrait of a Hamiltonian vector flow is . Which Hamiltonian is it generated from?

(A) $\mathcal{H} = p(x + p)$ (B) $\mathcal{H} = \frac{1}{2}\left(p^2 + 3x^2\right)$
(C) $\mathcal{H} = \frac{1}{2}\left(p^2 + x^2\right)$ (D) $\mathcal{H} = p(x - p)$
(E) $\mathcal{H} = \frac{1}{2}p^2 + (x^2 - 1)^2$ (F) $\mathcal{H} = \frac{1}{2}p^2 - (x^2 - 1)^2$

Canonical transformations I

Suppose that $Q = Q(q, p)$, $P = P(q, p)$ is a canonical transformation. If $Q = \sqrt{q}$, then P can be
(A) $P = 2p\sqrt{x}$ (B) $P = \sqrt{p}$
(C) $P = \frac{1}{\sqrt{p}}$ (D) $P = \sqrt{px} + h(x)$
(E) $P = \frac{p}{\sqrt{x}}$

For which value of β the transformation $P = q\,p$, $Q = \log\left(p^{\beta}\,q^{2014}\right)$ is canonical?
(A) 2013 (B) 2012
(C) 2014 (D) 2015
(E) any (F) none

For which value of β the transformation $P = q\,p$, $Q = \log\left(p^{2014}\,q^{\beta}\right)$ is canonical?
(A) 2015 (B) 2012
(C) 2014 (D) 2013
(E) any (F) none

Assuming $\cos\theta \neq 0$, for which value of α the transformation $P = \alpha\,\cos\theta\,p + \sin\theta q$, $Q = -\sin\theta\,p + \cos\theta\,q$ is canonical?

(A) 1
(B) -1
(C) 0
(D) any
(E) none

Canonical transformations II

The generating function which performs the canonical transformation $Q = p$, $P = -q$ is
(A) $+q\,Q$
(B) $-q\,Q$
(C) $q\,p$
(D) $-q\,p$
(E) this transformation is not canonical
(F) $Q\,P$

The physical meaning of the transformation generated by $S_3(p, Q) = -p\,Q + a\,p$ is
(A) shift of q by a
(B) shift of q by $-a$
(C) shift of q by $a\,p$
(D) shift of p by a
(E) this transform is not canonical

Let $q(t)$, $p(t)$ be a phase space trajectory of a 1-dimensional free particle. Define the following canonical transformation dependent on some parameter τ implicitly: $Q = q(\tau)$, $P = p(\tau)$, $q = q(0)$, $p = p(0)$. The generating function for this transformation is
(A) $S_1(q, Q) = -\frac{m}{2}\frac{(Q-q)^2}{\tau}$
(B) $S_1(q, Q) = \frac{m}{2}\frac{(Q-q)^2}{\tau^2}$
(C) $S_2(q, P) = \tau q\,P$
(D) $S_4(p, P) = \frac{\tau\,p^2}{2m}$

Hamilton-Jacobi

Q:When the Hamilton-Jacobi equation is interpreted as an eikonal approximation of a wave equation, the velocity of the particle is interpreted as

(A) group velocity of the wave
(B) phase velocity of the wave
(C) has no interpretation

Dispersion relation

Consider a wave equation in 1 time and 3 spartial dimensions: $\frac{\partial^2\Phi}{\partial t^2} - \sum_{i,j=1}^{3}\frac{\partial\Phi}{\partial x_i}\Omega_{ij}\frac{\partial\Phi}{\partial x_j}$. Dispersion relation reads:
(A) $\omega^2 = \sum_{i,j=1}^{3} k_i\Omega_{ij}k_j$
(B) $\omega = |k|\mathrm{tr}\,\Omega$
(C) There are three solutions, with dispersion relation $\omega = \Lambda_\alpha\,|k|$, Λ_α is an eigenvalue of Ω.

Q:Suppose that dispersion relation for a 3-dimensional wave is $\omega^2 = \sum_{i,j=1}^{3} k_i\Omega_{ij}k_j$ and Ω is symmetric positive-definite matrix. If the vector $\mathbf{k}$ is an eigenvector of Ω then

(A) Phase velocity and group velocity coincide
(B) Phase velocity and group velocity are parallel but do not have the same absolute value
(C) Phase velocity and group velocity are not parallel but have the same absolute value
(D) Phase velocity and group velocity do not coincide neither in direction nor in absolute value

Special relativity

Basic questions

Q:Which of the following statements is correct (if two are correct, choose the strongest one)?

(A) If in some reference frame A happens later than B and both A and B happen at the same point of space then A happens later than B in any reference frame
(B) If in some reference frame A happens later than B then A happens later than B in any reference frame
(C) If in some reference frame two space-time events happen in different points of space, they happen in different points of space in any reference frame
(D) If interval between two events is zero then these two events coincide
(E) Time inversion can be continuously transformed to the space inversion

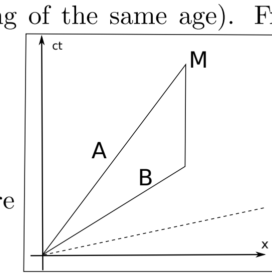
Q:The interval $\Delta s^2 > 0$ is called (if two are correct, choose the best):

(A) time-like
(B) space-like
(C) light-like
(D) positive

Q:The interval $\Delta s^2 < 0$ is called (if two are correct, choose the best):

(A) space-like
(B) time-like
(C) light-like
(D) negative

SR, basic computations
Length of rod at rest is $5m$. Rod moves with $\beta = 0.6$ with respect to an observer A. Observer A perceives the rod's length to be
(A) 4m (B) 6.25m
(C) 5m (D) 7.8m
(E) 3.2m
The proper life time of certain particle is $10^{-7}s$. The particle moves with $\beta = 0.6$ with respect to an observer A. From the point of view of the observer A, the particle will cover the following distance before decaying ($c = 3 \times 10^8 m/s$):
(A) 22.5m (B) 37.5m
(C) 24m (D) 30m
(E) 14.4m

Paradoxes
Q:Twin A and twin B start traveling from earth at the same time (and being of the same age). From the point of view of an inertial observer (e.g. Earth), their space-time trajectories are 

, where dashed line denotes propagation of light. When twins meet again at point M,

(A) The twin A is older
(B) The twin B is older
(C) Twins are of the same age
(D) Conclusion cannot be made from the provided information