The basics of Quantum Electrodynamics, and its role in Particle Physics

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Abstract

A review of quantum electrodynamics (QED) is presented along with its role in particle physics. Physical concepts will be emphasized rather than mathematical formalism. Feynman diagrams will be the main technique used.

Introduction

QED is the earliest of the quantum field theories (QFT's) of modern particle physics. It describes the interaction of electrons with photons (particles of light), and hence in theory describes all electromagnetic phenomena in the universe. A useful pictorial technique has been developed by Feynman [3] to describe QED, using *Feynman diagrams*, and generalizes to the other QFT's such as quantum chromo-dynamics (QCD) and electroweak theory.

The Basics

The *amplitude*, $\mathcal{M}$ for a process can be written down using the *Feynman rules.*
The *cross-section*, σ is the probability of an event happening.
The *differential cross-section* is defined as $\frac{d\sigma}{d\Omega}$, where Ω is the solid angle that particles are scattered into.

The defining relation is

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 \tag{1}$$

Equation (1) must be integrated to give σ. So given an interaction we eventually want to know its cross-section, which can be calculated using the above terms.

The amplitude of an event which can happen in more than one way is just the sum of the amplitudes for it to happen in each way, this is the principle of superposition in Quantum mechanics.

Feynman Diagrams

Feynman diagrams show particles moving and interacting in space-time. We define the vertical axis to be time and the horizontal axis to be space.
The three basic building blocks of any QED Feynman diagram are shown below.

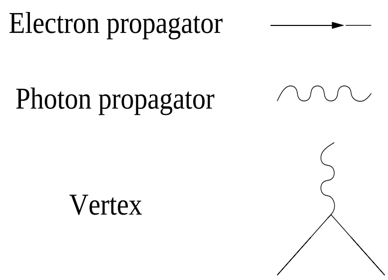

Figure 1: The three building blocks of all QED Feynman diagrams.

To each of these corresponds a mathematical expression, ascribed by the Feynman rules. Most importantly, for each vertex a factor of e (charge of electron) is inserted. Amplitudes will be written in power series expansions in terms of $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$ the *fine structure constant* or *electromagnetic coupling constant.* Thus each term will be α times less than the previous term, the small value of α makes expansions converge quickly. In practice only a few terms are required, and the leading term (*tree level*) is accurate to over 99%. Here we will work in tree level with a mention of loops in the last section.
There is only one vertex in QED, that with two fermions and one photon. *In QED, fermions can only interact by exchange of photons.*

Some simple QED processes

Møller scattering

The reaction $e^-e^- \rightarrow e^-e^-$ is called Møller scattering [4]. There are two Feynman diagrams for Møller scattering, shown in Figure 2.

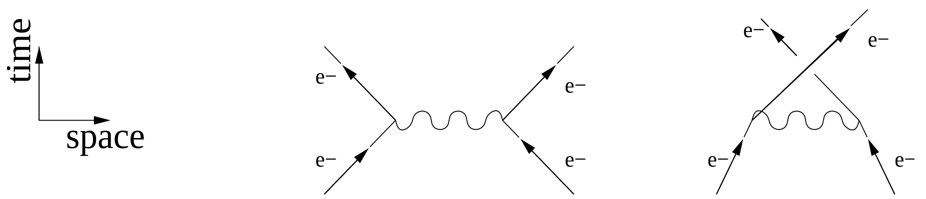

Figure 2: The two Feynman diagrams for Møller scattering.

Bhabha scattering

The reaction $e^+e^- \to e^+e^-$ is called Bhabha scattering [1]. There are two Feynman diagrams for Bhabha scattering, shown below.[1][2]

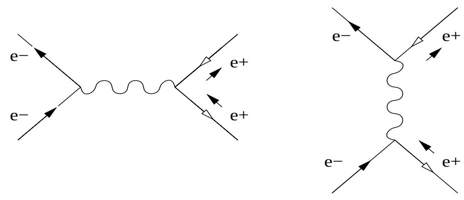

Figure 3: The two Feynman diagrams for Bhabha scattering.

Compton scattering

The reaction $e^-\gamma \to e^-\gamma$ is called Compton scattering [2]. Again there are two diagrams to describe it, shown in Figure 4.

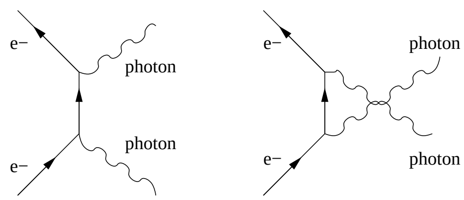

Figure 4: The two Feynman diagrams for Compton scattering.

Pair annihilation/creation

The process $e^+e^- \to \gamma\gamma$ is called pair annihilation, and the process $\gamma\gamma \to e^+e^-$ is called pair creation. Their diagrams are the same under time reversal.

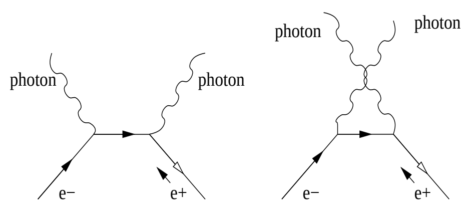

Figure 5: The two Feynman diagrams for pair annihilation.

Compton scattering and pair annihilation are related by *crossing symmetry.* This states that *the amplitude for any process involving a particle with momentum p in the initial state is equal to the amplitude for an otherwise identical process but with an antiparticle of momentum -p in the final state.* [5] This is very important in QFT.

[1] *e^+ is the positron, the antiparticle of the electron.*

[2] *It is conventional to flip directions of arrows on antiparticles, to allow interpretation of them moving forward in time (black arrows).*

Loops and Self Energies

Electron self energy

An electron propagating in a vacuum has corrections due to photons being radiated and absorbed. So experimentally what we see as an electron at point A and then at point B, could be that in Figure 6 or with a higher order loop.

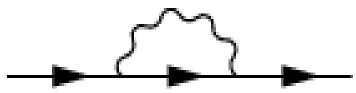

Figure 6: The one-loop correction to the electron self energy.

So the amplitude for an electron to propagate from A to B is the electron propagator plus the sum of all the loop diagrams. This means that physically a bare electron is surrounded by electron-positron pairs which screen the charge of the electron.

Photon self energy

Similarly, a photon propagating in a vacuum has corrections due to pair creation/annihilation.

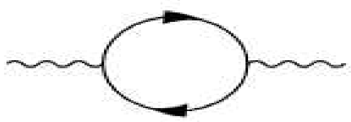

Figure 7: The one-loop correction to the photon self energy.

So the amplitude for a photon to propagate from A to B is the photon propagator plus the sum of all the loop diagrams.

Acknowledgments

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References

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