UNIVERSITY OF DUBLIN

XMA11241

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics
JF Two Subject Mod

Michaelmas Term 2012

MA1124 — ANALYSIS II

Thursday, September 06 REGENT HSE 9:30 — 12:00

Prof. D. O'Donovan

Answer all Questions.

All questions carry equal marks.

Formulae & Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. (a) Complete the following logical equivalences and use True/False tables to prove them

   i. $P \Rightarrow Q \equiv$

   ii. $(P \wedge Q)' \equiv$

   iii. $(P \wedge Q) \vee (R \wedge Q) \equiv$

   (b) Write down the negation of the following statements

   i. The relation R is reflexive and symmetric

   ii. If f is continuous and A is connected then $f(A)$ is connected.

2. (a) Define what it means for two sets to have the same cardinal number. Is there a set whose cardinal number is greater than that of any other set? Explain your reasoning.

   (b) Define what it means for one cardinal number to be greater or equal than another. Define what it means for a set to be countable. Prove that if X and Y are countable sets then $X \times Y$ is a countable set.

   (c) Define what it means for a subset of the real line to be connected. Show that a subset with just two points is not connected. What are the connected subsets of the real line?

3. (a) Prove that there exists a sequence $x_n \in A$ such that $\{x_n\} \to x$ if and only if for all $\epsilon > 0$, $N(x, \epsilon) \cap A \neq \emptyset$

   (b) Define liminf$\{a_n\}$, and limsup$\{a_n\}$. What is their relationship with $\lim\{a_n\}$?

   (c) Define $\overline{A}$, the closure of a set A, and A°, the interior of A. Prove $(A^\circ)^c = \overline{(A^c)}$.

   (d) Prove that if f is continuous and A is connected then $f(A)$ is connected.

4. (a) Prove the following for A a subset of $\mathbb{R}$: A is closed and bounded implies that every open covering of A has a finite subcovering.

   (b) Define what is meant by a Cauchy sequence and prove that every Cauchy sequence is bounded.

   (c) Prove that if a Cauchy sequence has a convergent subsequence, then the sequence converges