

Assignment 8 MA1124 March 14.

1. Prove that if  $a_n \rightarrow 0$  and  $\{b_n\}$  is a bounded sequence then  $\{a_n b_n\} \rightarrow 0$ .
2. Prove that  $\overline{A}$  is the smallest closed set of which  $A$  is a subset, and hence prove that  $\overline{A}$  is the intersection of all closed sets that contain  $A$ .
3. Prove that if  $f(x)$  is continuous at  $x = a$ , and  $f(a) \geq 0$ , then for some interval about  $x = a$ ,  $f(x)$  is positive.
4. Prove the corresponding fact about  $f(a) \leq 0$  as economically as possible. A nod to the TSM's?
5. Prove that  $f(x)$  is continuous at  $x = a$  if and only if for every sequence  $\{a_n\} \rightarrow a$  then  $f(x_n) \rightarrow f(a)$ .