MA1124 Assignment2

[due Monday 26 January, 2015]

1. Use the corresponding logical equivalence to prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. pages 98/99 22,24,26,28,44 These pages follow.
3. Define $x \sim y$ means 5 divides x-y for x and y integers. Show that this is an equivalence relation.
4. Define $(a, b) \sim (c, d)$ means a+d = b+c, on $Z^+ x Z^+$. Show that this is an equivalence relation.

c. $\exists$ a dessert D such that $\forall$ students S, S chose D.
d. $\exists$ a beverage B such that $\forall$ students D, D chose B.
e. $\exists$ an item I such that $\forall$ students S, S did not choose I.
f. $\exists$ a station Z such that $\forall$ students S, $\exists$ an item I such that S chose I from Z.

21. How could you determine the truth or falsity of the following statements for the students in your discrete mathematics class? Assume that students will respond truthfully to questions that are asked of them.
a. There is a student in this class who has dated at least one person from every residence hall at this school.
b. There is a residence hall at this school with the property that every student in this class has dated at least one person from that residence hall.
c. Every residence hall at this school has the property that if a student from this class has dated at least one person from that hall, then that student has dated at least two people from that hall.

Give the contrapositive, converse, and inverse of each statement in 22–29.

22. $\forall x \in \mathbf{R}$, if $x > 3$ then $x^2 > 9$.

23. $\forall$ computer programs P, if P is correct then P compiles without error messages.

24. If an integer is divisible by 6, then it is divisible by 3.

25. If the square of an integer is even, then the integer is even.

26. $\forall x \in \mathbf{R}$, if $x(x+1) > 0$ then $x > 0$ or $x < -1$.

27. $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$.

28. $\forall$ integers a, b, and c, if $a - b$ is even and $b - c$ is even, then $a - c$ is even.

29. $\forall$ animals A, if A is a cat then A has whiskers and A has claws.

30. Give an example to show that a universal conditional statement is not logically equivalent to its inverse.

Rewrite each statement of 31–34 in if-then form.

31. Earning a grade of C$-$ in this course is a sufficient condition for it to count toward graduation.

32. Being divisible by 6 is a sufficient condition for being divisible by 3.

33. Being on time each day is a necessary condition for keeping this job.

34. A grade-point average of at least 3.7 is a necessary condition for graduating with honors.

Use the facts that the negation of a $\forall$ statement is a $\exists$ statement and that the negation of an if-then statement is an *and* statement to rewrite each of statements 35–38 without using the words *sufficient* or *necessary*.

35. Divisibility by 4 is not a necessary condition for divisibility by 2.

36. Having a large income is not a necessary condition for a person to be happy.

37. Having a large income is not a sufficient condition for a person to be happy.

38. Being continuous is not a sufficient condition for a function to be differentiable.

39. The following statement is from *An Introduction to Programming.** Rewrite it without using the words *necessary* or *sufficient.*

The absence of error messages during translation o[f] a computer program is only a necessary and not [a] sufficient condition for reasonable [program] cor[rectness].

40. Find the answers Prolog would give if the followin[g] questions were added to the program given in Ex[ample] 2.2.9:
a. ?isabove(b_1, w_1)
b. ?isabove(w_1, g)
c. ?color(w_2, blue)
d. ?color(X, white)
e. ?isabove(X, b_1)
f. ?isabove(X, b_3)
g. ?isabove(g, X)

41. Write the negation of the definition of limit of a s[e]quence given in Example 2.2.3.

42. The notation $\exists!$ stands for the words "there exists unique." Thus, for instance, "$\exists!$ x such that x is pri[me] and x is even" means that there is one and only o[ne] even prime number. Which of the following stateme[nts] are true and which are false? Explain.
a. $\exists!$ real number x such that $\forall$ real numbers y, $xy =$
b. $\exists!$ integer x such that $1/x$ is an integer.
c. $\forall$ real numbers x, $\exists!$ real number y such t[hat] $x + y = 0$.

◆ 43. Suppose that $P(x)$ is a predicate and D is the domai[n of] x. Rewrite the statement "$\exists!$ $x \in D$ such that P[...]" without using the symbol $\exists!$. (See exercise 42 fo[r the] meaning of $\exists!$.)

*Richard Conway and David Gries, *An Introduction to Prog[ram]ming*, 2d ed. (Cambridge, Massachusetts: Winthrop, 1975), p.

♦44. Let $P(x)$ and $Q(x)$ be predicates and suppose D is the domain of x. For each pair of statements below, determine whether the statements have the same truth values. Justify your answers.

a. $\forall x \in D, (P(x) \land Q(x))$, and $(\forall x \in D, P(x)) \land (\forall x \in D, Q(x))$

b. $\exists x \in D, (P(x) \land Q(x))$, and $(\exists x \in D, P(x)) \land (\exists x \in D, Q(x))$

c. $\forall x \in D, (P(x) \lor Q(x))$, and $(\forall x \in D, P(x)) \lor (\forall x \in D, Q(x))$

d. $\exists x \in D, (P(x) \lor Q(x))$, and $(\exists x \in D, P(x)) \lor (\exists x \in D, Q(x))$

2.3 ARGUMENTS WITH QUANTIFIED STATEMENTS

The only complete safeguard against reasoning ill, is the habit of reasoning well; familiarity with the principles of correct reasoning; and practice in applying those principles. (John Stuart Mill)

The rule of **universal instantiation** (in-stan-she-AY-shun) says that

> If some property is true of *everything* in a domain, then it is true of *any particular* thing in the domain.

Use of the words *universal instantiation* indicates that the truth of a property in a particular case follows as a special instance of its more general or universal truth. The validity of this argument form follows immediately from the definition of truth values for a universal statement. One of the most famous examples of universal instantiation is the following:

All human beings are mortal.
Socrates is a human being.
∴ Socrates is mortal.

Universal instantiation is *the* fundamental tool of deductive reasoning. Mathematical formulas, definitions, and theorems are like general templates that are used over and over in a wide variety of particular situations. A given theorem says that such and such is true for all things of a certain type. If, in a given situation, you have a particular object of that type, then by universal instantiation, you conclude that such and such is true for that particular object. You may repeat this process 10, 20, or more times in a single proof or problem solution.

As an example of universal instantiation, suppose you are doing a problem that requires you to simplify

$$r^{k+1} \cdot r,$$

where r is a particular real number and k is a particular integer. You know from your study of algebra that the following universal statements are true:

1. For all real numbers x and all integers m and n, $x^m \cdot x^n = x^{m+n}$.
2. For all real numbers x, $x^1 = x$.

So you proceed as follows:

$$\begin{aligned} r^{k+1} \cdot r &= r^{k+1} \cdot r^1 && \text{step 1} \\ &= r^{(k+1)+1} && \text{step 2} \\ &= r^{k+2} && \text{by basic algebra.} \end{aligned}$$