**MA1124 Assignment1**
[due Monday 19 January, 2015]

Pages 13-17 8,13,41,48,50
Pages 27-28 14,16(d),24,37,39
Pages 39-41 6,8,26,29,39.
Pages 56-57 28,30,32 These pages follow.

0.

| P | Q | R | S |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

21.

| P | Q | R | S |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |

22. Design a circuit to take input signals $P$, $Q$, and $R$ and output a 1 if, and only if, $P$ and $Q$ have the same value and $Q$ and $R$ have opposite values.

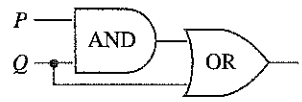
23. Design a circuit to take input signals $P$, $Q$, and $R$ and output a 1 if, and only if, all three of $P$, $Q$, and $R$ have the same value.

24. The lights in a classroom are controlled by two switches: one at the back and one at the front of the room. Moving either switch to the opposite position turns the lights off if they are on and on if they are off. Assume the lights have been installed so that when both switches are in the down position, the lights are off. Design a circuit to control the switches.

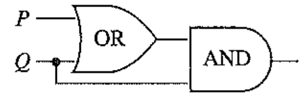
25. The ceiling light in an automobile is controlled by three switches: an automatic one in the driver's side door, another in the passenger's side door, and a manual one in the ceiling. Moving any switch to the opposite position turns the light off if it is on and on if it is off. Assume the light has been installed so that when the doors are closed and the ceiling switch is in the back position, the light is off. Design a circuit to control the switches.

Use the properties listed in Theorem 1.1.1 to show that each pair of circuits in 26–29 have the same input/output table. (Find the Boolean expressions for the circuits and show that they are logically equivalent, when regarded as statement forms.)

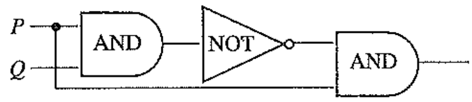
26. a.



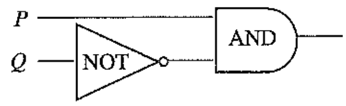

b.



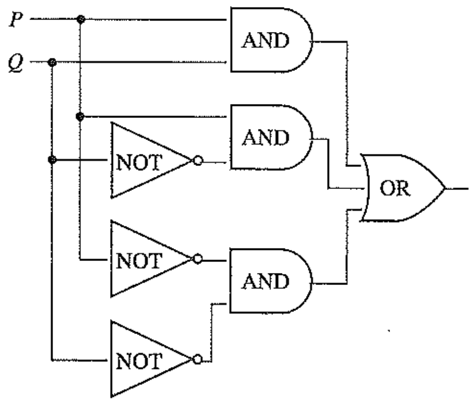

27. a.



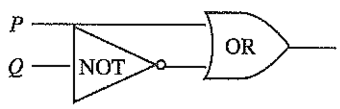

b.



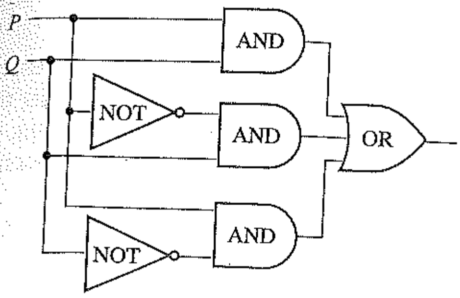

28. a.



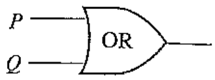

b.



29. a.




b.




**For the circuits corresponding to the Boolean expressions in each of 30 and 31 there is an equivalent circuit with at most two logic gates. Find such a circuit.**

**30.** $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

31. $(\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$

32. The Boolean expression for the circuit in Example 1.4.5 is

$$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$$

(a disjunctive normal form). Find a circuit with at most three logic gates that is equivalent to this circuit.

33. a. Show that for the Scheffer stroke $|$,

$$p \wedge q \equiv (p \mid q) \mid (p \mid q).$$

b. Use the results of Example 1.4.7 and part (a) above to write $p \wedge (\sim q \vee r)$ using only Scheffer strokes.

34. Show that the following logical equivalences hold for the Peirce arrow $\downarrow$, where $p \downarrow q \equiv \sim(p \vee q)$.

a. $\sim p \equiv p \downarrow p$ b. $p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q)$

c. $p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

*H* d. Write $p \rightarrow q$ using Peirce arrows only.

e. Write $p \leftrightarrow q$ using Peirce arrows only.

# 1.5 APPLICATION: NUMBER SYSTEMS AND CIRCUITS FOR ADDITION

*Counting in binary is just like counting in decimal if you are all thumbs.*
(Glaser and Way)

In elementary school, you learned the meaning of decimal notation: that to interpret a string of decimal digits as a number, you mentally multiply each digit by its place value. For instance, 5,049 has a 5 in the thousand's place, a 0 in the hundred's place, a 4 in the ten's place, and a 9 in the one's place. Thus,

$$5{,}049 = 5 \cdot (1{,}000) + 0 \cdot (100) + 4 \cdot (10) + 9 \cdot (1).$$

Using exponential notation, this equation can be rewritten as

$$5{,}049 = 5 \cdot (10^3) + 0 \cdot (10^2) + 4 \cdot (10^1) + 9 \cdot (10^0).$$

More generally, decimal notation is based on the fact that any positive integer can be written uniquely as a sum of products of the form

$$d \cdot (10^n),$$

where each $n$ is a nonnegative integer and each $d$ is one of the decimal digits 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. The word *decimal* comes from the Latin root *deci,* meaning "ten." Decimal (or base 10) notation expresses a number as a string of digits in which each digit's position indicates the power of 10 by which it is multiplied. The right-most position is the one's place (or $10^0$ place), to the left of that is the ten's place (or $10^1$ place), to the left of that is the hundred's place (or $10^2$ place), and so forth, as illustrated below.

| place | $10^3$ thousands | $10^2$ hundreds | $10^1$ tens | $10^0$ ones |
|---|---|---|---|---|
| decimal digit | 5 | 0 | 4 | 9 |