Assignment 9 Solutions

p448.

$$\lim_{x \to 0^+} x^x = ?$$

$$\ln x^x = x \ln x \to 0 \cdot -\infty$$
$$= \frac{\ln x}{1/x} \to \frac{-\infty}{\infty}$$
$$\overset{\mathcal{L}}{=} \frac{1/x}{-1/x^2} = x \to 0.$$

$$\therefore e^{\ln x^x} = x^x \to e^0 = 1$$

since e^x is continuous all x.

p449.

(a) $$\lim_{x \to 0^+} x^{\ln a/(1+\ln x)}$$

$$\ln x^{\ln a/(1+\ln x)} = \frac{\ln a}{1+\ln x} \cdot \ln x$$
$$= \ln a \cdot \frac{1}{\frac{1}{\ln x}+1} \to \ln a \cdot \frac{1}{0+1}$$
$$= \ln a$$

or use L'Hopital

$$\therefore e^{\ln x^{\ln a/(1+\ln x)}} \to e^{\ln a} = a$$

since e^x is cont.

(b) identical.

(c) $\lim_{x\to 0} (x+1)^{\ln a/x}$

$\ln (x+1)^{\ln a/x} = \frac{\ln a}{x} . \ln(x+1)$

$\to \ln a . \frac{0}{0}$

$\stackrel{\mathcal{L}}{=} \ln a . \frac{\frac{1}{x+1}}{1}$

$\to \ln a . \frac{1}{0+1} = \ln a$

$\therefore \quad (x+1)^{\ln a/x} \to a.$

since e^x is cont.

5 8 $\lim_{x\to\infty} \frac{2x - \sin x}{3x + \sin x} \to \frac{\infty}{\infty}$

$\stackrel{\mathcal{L}}{=} \frac{2 - \cos x}{3 + \cos x} \to$ No limit, cos oscillates

But $\frac{2x - \sin x}{3x + \sin x} = \frac{2 - \frac{\sin x}{x}}{3 + \frac{\sin x}{x}}$

$\to \frac{2 - 0}{3 + 0} = 2/3$

p 484

$$T(t) = 400 - 325(0.97)^t$$

$$260 = 400 - 325(0.97)^{t_1}$$

$$140 = 325(0.97)^{t_1}$$

$$(0.97)^{t_1} = \frac{140}{325}$$

$$t_1 \ln(0.97) = \ln\left(\frac{140}{325}\right)$$

$$\therefore t_1 = \ln\left(\frac{140}{325}\right) / \ln(0.97)$$

$$= 27.65$$

Sim $t_2 = 32.71$

(b) $T(0) = 400 - 325.1$

$= 75$

$\therefore$ Initial difference $= 325$

$\frac{1}{2} = 162.5$

This occurs when $T = 237.5$

$$237.5 = 400 - (325)(.97)^{t_3}$$

Solve $t_3 = 22.76$

18 $\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^{bx}$. $a, b > 0$

$$\ln\left(1 + \frac{a}{x}\right)^{bx} = bx \ln\left(1 + \frac{a}{x}\right)$$

$$\to \infty \cdot 0$$

$$= b \, \frac{\ln(1 + a/x)}{1/x}$$

$$\overset{L}{=} b \, \frac{\frac{1}{1+a/x} \cdot \frac{-a}{\cancel{x^2}}}{-\frac{1}{\cancel{x^2}}}$$

$$= ab \cdot \frac{1}{1 + \frac{a}{x}} \to ab.$$

$$\therefore \left(1 + \frac{a}{x}\right)^{bx} \to e^{ab}.$$

p 485

23 $y = (\ln x)^2$

$$\frac{dy}{dx} = 2 \ln x \cdot \frac{1}{x}.$$

34. $y = 2^{\sin^{-1} x}$ $\qquad y = a^x \quad \frac{dy}{dx} = a^x \ln a$

$$\frac{dy}{dx} = 2^{\sin^{-1} x} \cdot \ln 2 \cdot \frac{d(\sin^{-1} x)}{dx}$$

$$= 2^{\sin^{-1} x} \cdot \ln 2 \cdot \frac{1}{\sqrt{1 - x^2}}$$

40. $y = \sqrt[3]{\dfrac{x^2-1}{x^2+1}}$

$\ln y = \frac{1}{3} \ln\left(\dfrac{x^2-1}{x^2+1}\right)$

$= \frac{1}{3} \ln(x^2-1) - \ln(x^2+1)$

$\dfrac{1}{y} \dfrac{dy}{dx} = \frac{1}{3}\left[\dfrac{1}{x^2-1} \cdot 2x - \dfrac{1}{x^2+1} \cdot 2x\right].$

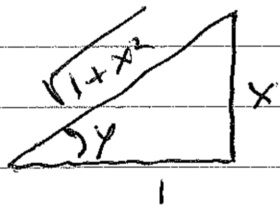
$\dfrac{dy}{dx} = \frac{1}{3} \sqrt{\dfrac{x^2-1}{x^2+1}} \left[\dfrac{2x}{x^2-1} - \dfrac{2x}{x^2+1}\right]$

52. $y = \tan^{-1} x$ $\qquad x = \tan y.$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

$\dfrac{d^2y}{dx^2} = -\dfrac{1}{(1+x^2)^2} \cdot 2x$

$= -\dfrac{2x}{(1+x^2)^2}$



$= -2 \dfrac{x}{\sqrt{1+x^2}} \cdot \dfrac{1}{(\sqrt{1+x^2})^3}$

$= -2 \sin y \cos^3 y.$