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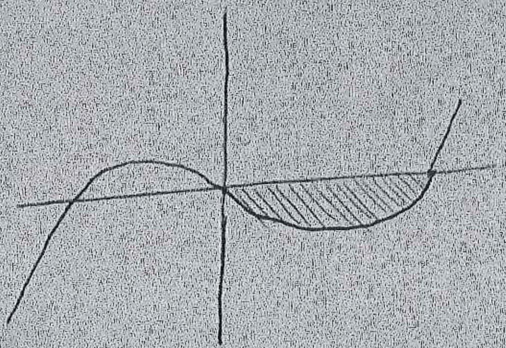
Q8. $y = x^3 - 4x$, $y = 0$, $x = 0$, $x = 2$

$$I = \int_0^2 -(x^3 - 4x)\,dx$$



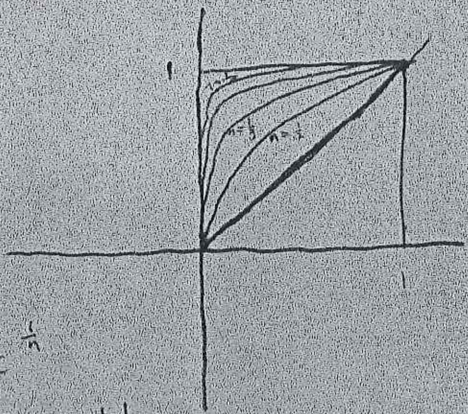
$$= \left[-\frac{x^4}{4} + 2x^2\right]_0^2$$

$$= [-4 + 8] - [0]$$

$$= +4.$$

pg 354

Q38.



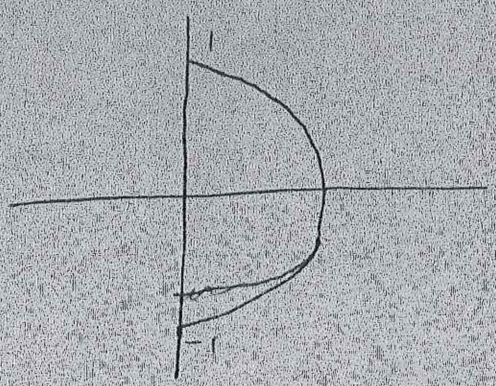
a) Observing how the graph of $x^{\frac{1}{n}}$ changes as n increases we would ~~set~~ conjecture $\lim_{n\to\infty} A(n) = \frac{1}{2}$

b) $I = \int_0^1 x^{\frac{1}{n}}\,dx = \left[\frac{n\,x^{\frac{n+1}{n}}}{n+1}\right]_0^1 = \frac{n}{n+1} - 0$

$\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$, $\int_0^1 x\,dx = \frac{1}{2}$

$\Rightarrow \lim_{n\to\infty} A(n) = \frac{1}{2}$ ▨

Q16



Cross-sections perpendicular to the y axis are squares

$\Rightarrow$

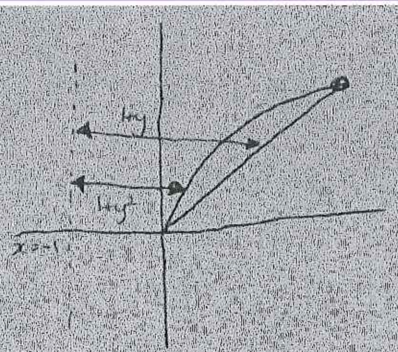
$$I = \int_{-1}^{1} (1-y^2)^2 \, dy$$

$$= \int_{-1}^{1} (1 - 2y^2 + y^4) \, dy$$

$$= \left[y - \frac{2}{3}y^3 + \frac{y^5}{5} \right]_{-1}^{1} = \left[1 - \frac{2}{3} + \frac{1}{5} \right] - \left[-1 + \frac{2}{3} - \frac{1}{5} \right]$$

$$= \frac{16}{15}.$$

Q36.



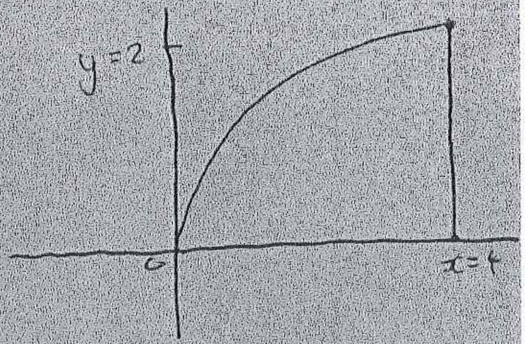
$$V(y) = \pi \int_0^1 (1+y)^2 - (1+y^2)^2 \, dy$$

$$= \pi \int_0^1 \left(2y - y^2 - y^4 \right) dy$$

$$= \pi \left[y^2 - \frac{y^3}{3} - \frac{y^5}{5} \right]_0^1 = \pi \left[1 - \frac{1}{3} - \frac{1}{5} \right]$$

$$= \frac{7\pi}{15} .$$

Q 42.



$$V = \frac{\pi}{2} \int_0^4 \left(\frac{\sqrt{x}}{2} \right)^2 dx$$

$$= \frac{\pi}{2} \int_0^4 \frac{x}{4} \, dx = \frac{\pi}{8} \left[\frac{x^2}{2} \right]_0^4 = \pi$$

$$\boxed{V = \pi}$$

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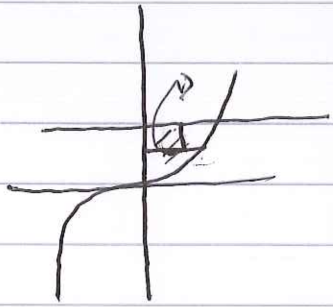

$$\int_0^1 y^{1/3} \cdot 2\pi(1-y)\,dy$$

$$= 2\pi \int_0^1 y^{1/3} - y^{4/3}\,dy$$

$$= 2\pi \left. \frac{y^{4/3}}{4/3} - \frac{y^{7/3}}{7/3} \right|_0^1$$

$$= 6\pi\left(\frac{1}{4} - \frac{1}{7}\right) = \frac{3}{28}$$

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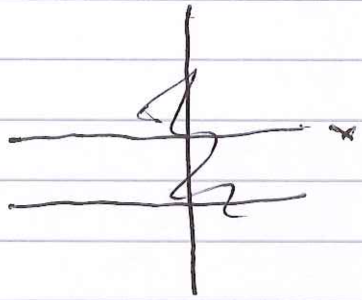
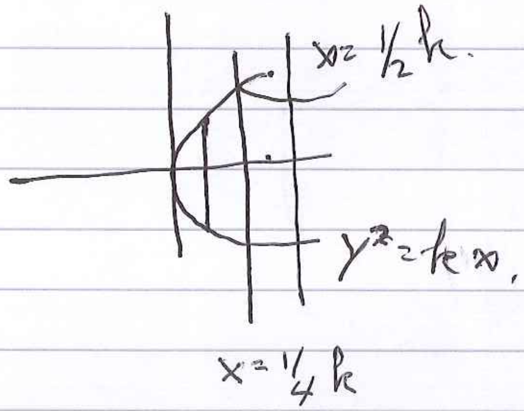




$$\int_0^{\frac{1}{4}k} \left(\sqrt{kx} - (-\sqrt{kx})\right) 2\pi\left(\tfrac{1}{2}k - x\right) dx$$

$$= \cancel{4\pi} \int_0^{\frac{1}{4}k} \tfrac{1}{2}k\sqrt{kx} - x\sqrt{kx}\;dx$$

$$= \cancel{4\pi} \left[\tfrac{1}{2}k^{1/2} \frac{x^{3/2}}{3/2} - k^{1/2} \frac{x^{5/2}}{5/2} \right]_0^{\frac{1}{4}k} = \cancel{4\pi}\left[\frac{k^3}{24} - \frac{1k^3}{32.5}\right]$$

375-4) $L = \int_0^1 \left[1+\left(\frac{dx}{dy}\right)^2\right]^{1/2} dy$, $\frac{dx}{dy} = y(y^2+2)^{1/2}$

$\Rightarrow L = \int_0^1 [1+y^2(y^2+2)]^{1/2} dy = \int_0^1 [y^4+2y^2+1]^{1/2} dy = \int_0^1 (y^2+1)\, dy$

$= \left[\frac{y^3}{3}+y\right]_{y=0}^{1} = \frac{1}{3}+1 = 4/3$

375-9) For checking whether $y = \sqrt{1-x^2}$ is smooth or not on $[-1,1]$, let us check whether dy/dx is continuous or not on $[-1,1]$

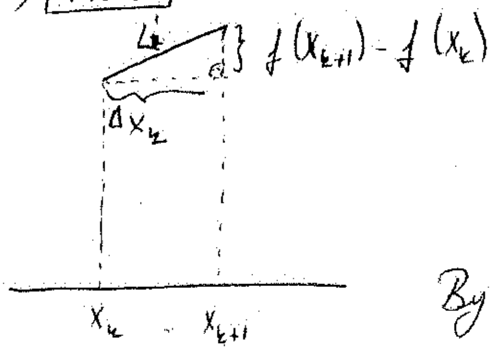
$f'(x) = \frac{dy}{dx} = -\frac{x}{\sqrt{1-x^2}}$, $f'(x)$ has not continuous on $[-1,1]$, hence it is not smooth curve.

$\boxed{\text{FALSE}}$

375-10) $\boxed{\text{TRUE}}$

For Riemann sum form, Mean-Value Theorem has to be applied to the expression. It leads equal small interval in x.

376-11) $\boxed{\text{TRUE}}$



Let L_k' be the real length of the line segment. By Pythagorean theorem

$$L_k = \left\{[f(x_k+1) - f(x_k)]^2 + \Delta x_k^2\right\}^{1/2}$$

By approx. formula

$$L_k = \left\{1+[f'(x_k^*)]^2\right\}^{1/2} \Delta x_k$$

$$f'(x_k) = \frac{f(x_{k+1}) - f(x_k)}{\Delta x_k}$$ for linear fun. / no limit

$\Rightarrow L_k = L_k' \Rightarrow$ Approx. become exact.

375-12) $\boxed{\text{FALSE}}$

$f(x)$ itself has to be continuous on $[a,b]$ to be able to apply MVT in arclength definition.

$f'(x)$ has to be continuous on $[a,b]$ to be able to evaluate the integral. It is put to guarantee the integrability.

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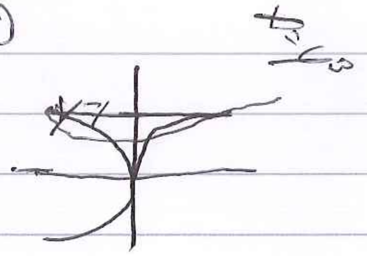
#6



$$\int_0^1 2\pi y^3 \sqrt{1+(3y^2)^2}\, dy.$$

$$= 2\pi \int_0^1 y^3 \sqrt{1+9y^4}\, dy.$$

Let $u = 1 + 9y^4$

$$\frac{du}{dy} = 36y^3.$$

$$= 2\pi \int \frac{1}{36} \frac{du}{dy} \sqrt{u}\, dy$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} \cdot u^{3/2}$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} \cdot (1+9y^4)^{3/2} \Big|_0^1$$

$$= \frac{\pi}{27}\left[10^{3/2} - 1\right].$$

#8

$$\int_{-1}^0 2\pi\, 2\sqrt{1-y}\, \sqrt{1 + \left(\frac{-1}{\sqrt{1-y}}\right)^2}\, dy.$$

$$= 4\pi \int_{-1}^0 \sqrt{1-y}\, \sqrt{1 + \frac{1}{1-y}}\, dy.$$

$$= 4\pi \int_{-1}^0 \sqrt{1-y}\, \frac{\sqrt{2-y}}{\sqrt{1-y}}\, dy$$

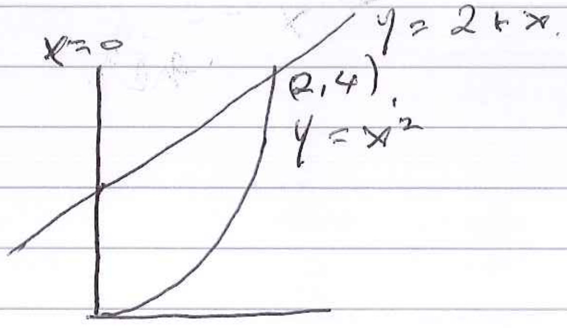
$$= 4\pi \int_{-1}^0 \sqrt{2-y}\, dy.$$

$$= 4\pi \cdot -1 \cdot \frac{2}{3} \cdot (\sqrt{2-y})^3 \Big|_{-1}^0$$

$$= -\frac{8\pi}{3}\left[(\sqrt{2})^3 - (\sqrt{3})^3\right]$$

$$= \frac{8\pi}{3}\left[(\sqrt{3})^3 - (\sqrt{2})^3\right]$$

407 # 6.



(a) Area $= \int_0^2 2+x-x^2\,dx.$

(b) Area $= \int_0^2 \sqrt{y}\,dy + \int_2^4 \sqrt{y}-(y-2)\,dy$

(c) Volume about x-axis $= \int_0^2 \pi(2+x)^2 - \pi(x^2)^2\,dx$

(d) Volume about x-axis $= \int_0^2 \sqrt{y}.\,2\pi y\,dy + \int_2^4 (\sqrt{y}-(y-2))2\pi y\,dy.$

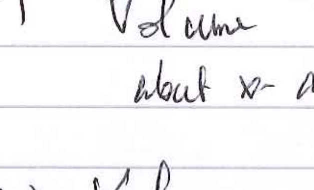
By cylindrical slices

(e) Volume about y-axis $= \int_0^2 (2+x-x^2)\,2\pi x\,dx.$

By cylindrical slices.

(f) $= \int_0^2 \pi(\sqrt{y})^2\,dy + \int_2^4 \pi(\sqrt{y})^2 - \pi(y-2)^2\,dy$

(g) Volume about $y=-3$ $\int_0^2 \pi(2+x+3)^2 - \pi(x^2+3)^2\,dx.$



(h) Volume about $x=5$ $\int_0^2 (2+x-x^2)\,2\pi(5-x)\,dx.$