Assignment 7
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21 True $h(x) = c$ $\int_a^b c\,dx = c(b-a)$

22 False $\int_a^b f(x) - g(x)\,dx = -3$
$\not\Rightarrow A = 3$

$f(x)$ + $g(x)$ could cross many times

23 True if $\int_c^b f(x) - g(x)\,dx = 0$.

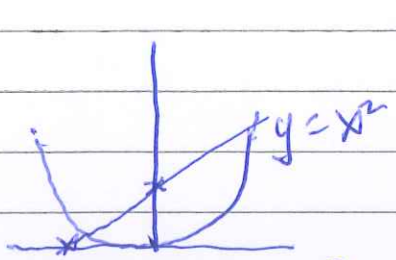
then $f(x) - g(x) = 0 \ \forall x$ - cross everywhere
or $f(x) - g(x) > 0$ somewhere
$\Rightarrow f(x) - g(x) < 0$ "
$\Rightarrow f(x) = g(x)$ somewhere
and they cross.

24 True if $A = \left| \int f(x) - g(x)\,dx \right|$

then $f(x) - g(x) \geq 0 \ \forall x$
or $f(x) - g(x) \leq 0 \ \forall x$.

$A =$ the ~~sum of~~ $\int_a^b |f(x) - g(x)|\,dx$

12.

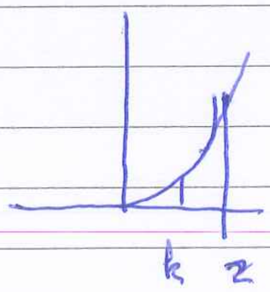


Points of intersection
$x^2 = x + 2$
$x^2 - x - 2 = (x-2)(x+1) = 0$
$x = -1, 2$

$$\text{Area} = \int_{-1}^{2} x + 2 - x^2\,dx = \frac{x^2}{2} + 2x - \frac{x^3}{3} \Big|_{-1}^{2}$$

$$= 6 - \tfrac{8}{3} - \left(-\tfrac{3}{2} + \tfrac{1}{3}\right) = \cancel{4}\tfrac{1}{2} = 4\tfrac{1}{2}$$

28



Want $\int_0^k x^2\,dx = \int_k^2 x^2\,dx$

$$\frac{k^3}{3} = \frac{8}{3} - \frac{k^3}{3}.$$

$$2k^3 = 8$$

$$k = \sqrt[3]{4}.$$

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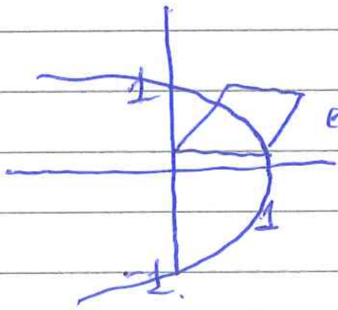

← square of side $1-y^2$

Area $= (1-y^2)^2$

$$\text{Volume} = \int_{-1}^{1} (1-y^2)^2\,dy$$

$$= \int_{-1}^{1} 1 - 2y^2 + y^4\,dy = y - \frac{2}{3}y^3 + \frac{y^5}{5}\Big|_{-1}^{1}$$

$$= 1 - 2/3 + 1/5 - (-1 + 2/3 - 1/5)$$

$$= 2 - 4/3 + 2/5 = 2/3 + 2/5 = 16/15$$

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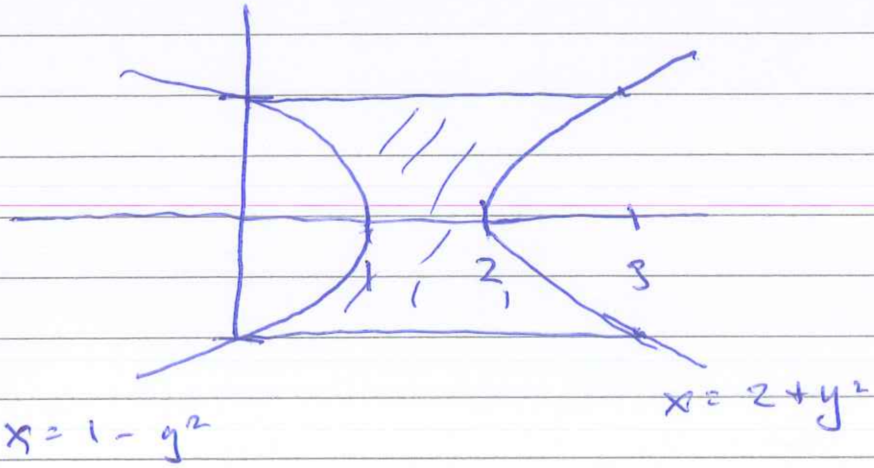

When this region is rotated about the y-axis the cross section is a washer of inner radius $1-y^2$ and outer radius $2+y^2$

So Volume $= \int_{-1}^{1} \pi(2+y^2)^2 - \pi(1-y^2)^2 \, dy$

$= \pi \int_{-1}^{1} 4+4y^2+y^4 - (1-2y^2+y^4) \, dy$

$= \pi \int_{-1}^{1} 3 + 6y^2 \, dy = \pi(3y+2y^3)\Big|_{-1}^{1}$

$= \pi(5-(-3-2)) = 10\pi$.

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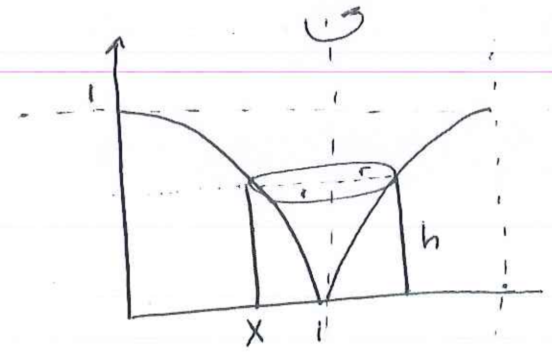
Q 26)

a)



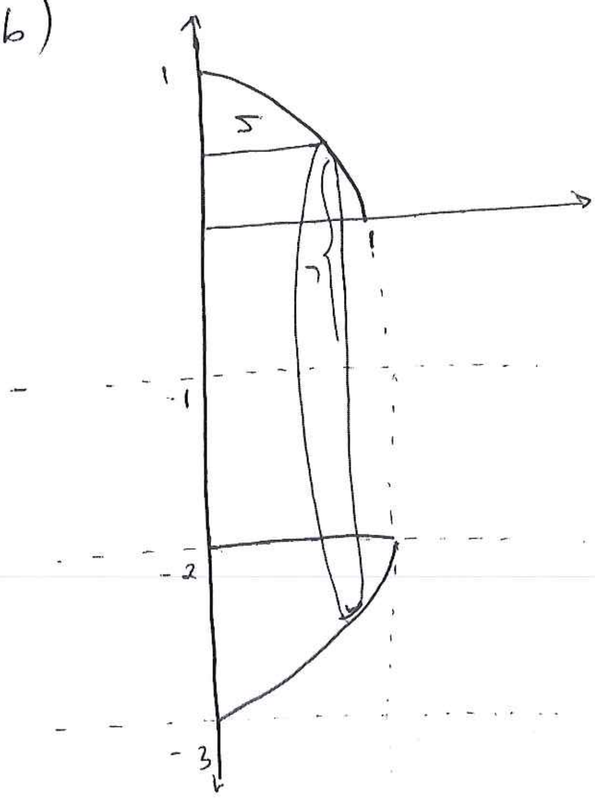
$r = (1-x)$

$h = \sqrt{1-x^2}$

$A(x) = 2\pi(1-x)\sqrt{1-x^2}$

$$V = \int_0^1 2\pi(1-x)\sqrt{1-x^2}\,dx$$

b)



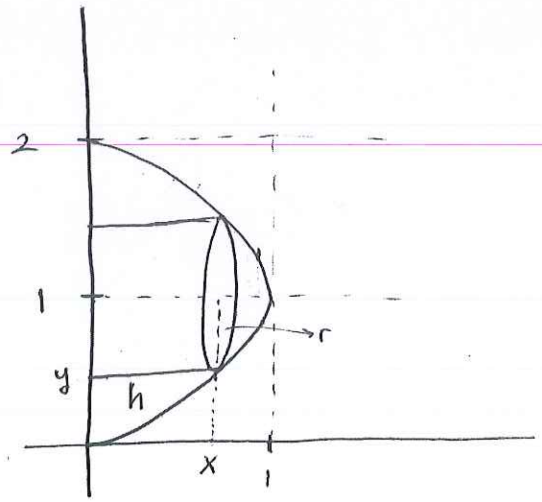
$r = y+1$

$h = \sqrt{1-y^2}$

$A(x) = 2\pi(y+1)\sqrt{1-y^2}$

$$V = \int_0^1 2\pi(y+1)\sqrt{1-y^2}\,dy$$

Q28)



$r = 1-y$

$h = x = \sqrt[3]{y}$

$A(y) = 2\pi(1-y)\sqrt[3]{y}$

$V = \int_0^1 2\pi(1-y)\sqrt[3]{y}\,dy$

$= 9\pi/14$

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Q9) $\boxed{\text{False}}$

$f'(x) = -\dfrac{x}{\sqrt{1-x^2}}$ is not continuous on $[-1,1]$

Q10) $\boxed{\text{True}}$

MVT shoul be applied to the term:

$[f(x_k) - f(x_{k-1})]^2$ to obtain the form of a Riemann sum.

Q11) $\boxed{\text{True}}$

Each L_k becomes exact arclength.

Q12) $\boxed{\text{False}}$

This requirement applies to $f(x)$.

Q26)

$$L = \int_0^1 \left[\left[2(1+t) \right]^2 + \left[3(1+t)^2 \right]^2 \right]^{1/2} dt$$

$$= \int_0^1 (1+t)\sqrt{4 + 9(1+t)^2}\, dt \,, \quad u = (1+t)^2$$

$$\frac{du}{dt} = 2(1+t)$$

$$t = 0 \Rightarrow u = 1$$

$$t = 1 \Rightarrow u = 4$$

$$= \frac{1}{2} \int_1^4 \sqrt{4+9u}\, du$$

$$= \frac{1}{2} \cdot \frac{1}{9} \cdot \frac{2}{3} \left(4+9u\right)^{3/2} \Big|_{u=1}^{4} \approx 7.6337$$

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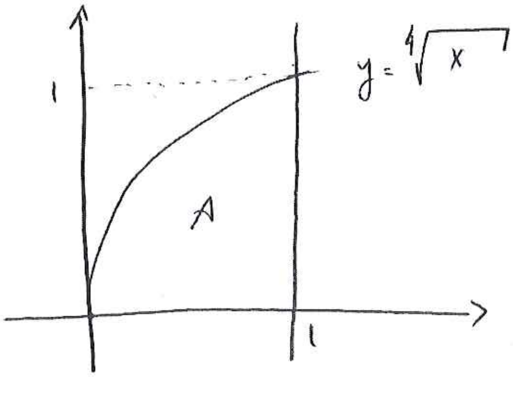
Q29) 22



$$A = \int_0^1 \sqrt[4]{x}\, dx = 4/5$$

$$M = \delta \cdot A = 15 \cdot 4/5 = 12 \text{ unit of mass}$$

δ : constant

$$\bar{x} = \frac{1}{A} \int_0^1 x \cdot \sqrt[4]{x}\, dx = \frac{5}{4} \cdot \frac{4}{9} = 5/9$$

$$\bar{y} = \frac{1}{A} \int_0^1 \frac{1}{2} \sqrt{x}\, dx = \frac{5}{4} \cdot \frac{1}{2} \cdot \frac{2}{3} = 5/12$$

Analysis Solutions 7

Problem (363: 21-24). *(Solid S of volume V is bounded by two parallel planes at $x = a$ and $x = b$ with cross sectional area $A(x)$.)*

If each cross section of S perpendicular to the x axis is a square then S is a rectangular paralleliped.

If each cross section of S is a disk or washer, then S is a volume of revolution.

If x is in centimeters, then $A(x)$ must be a quadratic function of x.

The average value of $A(x)$ on the interval $[a, b]$ is $V/(b - a)$.

Solution: False, S could be a square based pyramid with axis of symmetry on the x-axis.

False. Consider a solid whose cross sectional area to x-axis is a circle, but the centre of each circle lies on the line $y = x$. This is also not a solid of revolution around $y = x$, otherwise the cross sectional area perpendicular to the x axis would be elliptic.

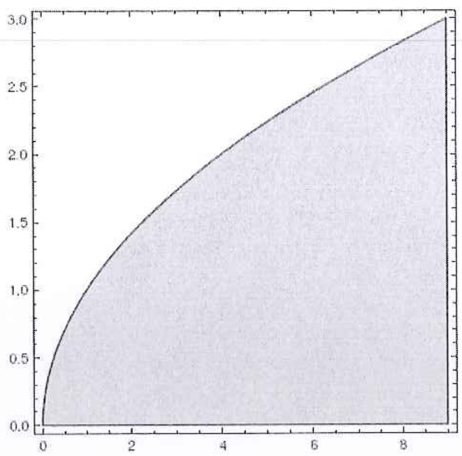
False. The units of $A(x)$ are dependent upon the units of the y axis, also see examples in the book where A is a linear function etc.

True. Since $V = \int_a^b A(x)\mathrm{d}x$, this is true by the MVT of integration.
□

Problem (34). *Find the volume of the solid when the region enclosed by $y = \sqrt{x}, y = 0, x = 9$ is revolved about the line y=3.*

Solution: □ The region is shown below, and we're to revolve about $y = 3$. This is best done using the "washer" method. The outer radius is $r_2 = 3$ and the inner is $r_1 = 3 - \sqrt{x}$:

$$\begin{aligned} V =& \pi \int r_2(x)^2 - r_2(x)^2 \mathrm{d}x \\ =& \pi \int_0^9 3 - (3 - \sqrt{x}^2)\mathrm{d}x \\ =& 135\pi/2 \end{aligned}$$



False. Section 5.2 outlines how to find the volume given the cross sectional area, cylindrical shells does not use cross sectional area.

True.

True. If $f = c$ is a constant then the fact that this is exact is equivalent to

$$V = c\sum_{k=1}^{n} 2\pi x_k^* \Delta x_k = c\int_a^b 2\pi x \mathrm{d}x$$

$$\sum_{k=1}^{n} \frac{(x_k - x_{k-1})}{2}(x_k - x_{k+1}) = \frac{(b^2 - a^2)}{2}$$

$$= \sum_{k=1}^{n} \frac{(x_k^2 - x_{k-1}^2)}{2})$$

The series on the left is telescoping, so only the first and last term contribute $= b^2 - a^2/2$. □