ASSIGNMENT #6 - Solution

Q1 $\int \sin^2 x \cos^4 x \, dx$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\cos^2 2x = \frac{1}{2}(1+\cos 4x)$

$$\Rightarrow \int \frac{1}{8}(1-\cos 2x)(1+\cos 2x)^2 \, dx$$

$$= \frac{1}{8}\int (1 + \cos 2x - \cos^2 2x - \cos^3 2x) \, dx$$

$$= \frac{1}{8}\int \left(\frac{1}{2} + \cos 2x - \frac{1}{2}\cos 4x - \frac{1}{2}\cos 2x - \frac{1}{2}\cos 2x \cos 4x\right) dx$$

$$= \frac{1}{8}\int \left(\frac{1}{2} + \frac{1}{4}\cos 2x - \frac{1}{2}\cos 4x - \frac{1}{4}\cos 6x\right) dx$$

$$= \frac{x}{16} + \frac{\sin 2x}{64} - \frac{\sin 4x}{64} - \frac{\sin 6x}{192} + C$$

Q2 $I = \int \frac{\sin\left(\frac{5}{x}\right)}{x^2} \, dx$, $u = \frac{5}{x} \Rightarrow \frac{du}{dx} = \frac{-5}{x^2}$

$$\Rightarrow -\frac{du}{5} = \frac{dx}{x^2}$$

$$I = -\frac{1}{5}\int \sin u \, du = \frac{1}{5}\cos u + C$$

$$= \frac{1}{5}\cos\left(\frac{5}{x}\right) + C$$

Q3. $I = \int x^2 \sqrt{1+x} \, dx$, $u = 1+x$ $dx = du$

$$\Rightarrow x^2 = (u-1)^2$$

$$\Rightarrow x^2\sqrt{1+x} = (u^2 - 2u + 1)u^{\frac{1}{2}}$$

$$= u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}$$

$$I = \int \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du = \frac{2}{7}u^{\frac{7}{2}} - \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C$$

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48. $p'(t) = (3 + 0.12t)^{\frac{3}{2}}$, $p(0) = 100$

$u = 3 + 0.12t = 3 + \frac{3}{25}t \Rightarrow du = \frac{3}{25}dt$

$\Delta P \, \cancel{p(t)} = \int_0^5 (3 + 0.12t)^{\frac{3}{2}} dt$

$= \frac{25}{3} \int_3^{\frac{18}{5}} u^{\frac{3}{2}} du = \frac{25}{3} \left[\frac{2}{5} u^{\frac{5}{2}} \right]_3^{\frac{18}{5}}$

$= \frac{10}{3} \left[\left(\frac{18}{5}\right)^{\frac{5}{2}} - (3)^{\frac{5}{2}} \right] = (9.001)\frac{10}{3} = 30$

$\Rightarrow P(5) = 130 \times 10^{3}$ minnows

Q4 $\displaystyle\int \frac{x}{\sqrt{2x+1}}\,dx$, $u = 2x+1$

$\frac{1}{2}\,du = dx$

$$\int \frac{x}{\sqrt{2x+1}}\,dx = \int \frac{\frac{1}{2}(u-1)}{u^{\frac{1}{2}}}\,\frac{du}{2}$$

$$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du = \frac{1}{4}\left[\frac{2u^{\frac{3}{2}}}{3} - 2u^{\frac{1}{2}}\right] + C$$

$$= \frac{1}{6}u^{\frac{3}{2}} - \frac{1}{2}u^{\frac{1}{2}} + C = \frac{1}{6}(2x+1)^{\frac{3}{2}} - \frac{1}{2}(2x+1)^{\frac{1}{2}} + C$$

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44. $\frac{dy}{dx} = 2 + \sin 3x$, $y\left(\frac{\pi}{3}\right) = 0$

$$y = \int (2 + \sin 3x)\,dx$$

$$= 2x - \frac{1}{3}\cos 3x + C$$

$$y\left(\frac{\pi}{3}\right) = \frac{2\pi}{3} - \frac{1}{3}\cos \pi + C = 0$$

$$\Rightarrow C = -\frac{1}{3} - \frac{2\pi}{3}$$

$$y(x) = 2x - \frac{1}{3}\cos 3x - \frac{1}{3} - \frac{2\pi}{3}.$$

18) a) $f(x) = 2x \quad ; \quad 0 \leq x \leq 1$

$$\Rightarrow \int_0^1 f(x)\,dx = \int_0^1 2x\,dx = (x^2+c)\Big|_0^1 = 1+c-0-c = 1$$

b) $f(x) = 2x \quad ; \quad -1 \leq x \leq 1$

$$\int_{-1}^{1} 2x\,dx = x^2\Big|_{-1}^{1} = 1-(1) = 0$$

c) $f(x) = 2 \quad ; \quad 1 < x \leq 10$

$$\int_1^{10} 2\,dx = (2x)\Big|_1^{10} = 20-2 = 18$$

d) $f(x) = 2x$ for $1/2 \leq x \leq 1$

$f(x) = 2$ for $1 < x \leq 5$

$$\int_{1/2}^{5} f(x)\,dx = \int_{1/2}^{1} f(x)\,dx + \int_1^5 f(x)\,dx = \int_{1/2}^{1} (2x)\,dx + \int_1^5 2\,dx$$

$$= x^2\Big|_{1/2}^{1} + 2x\Big|_1^5 = (1-1/4) + (10-2) = 3/4 + 8 = 35/4$$

24)

$$\int_{3}^{-2} f(x)\,dx = \int_{3}^{1} f(x)\,dx + \int_{1}^{-2} f(x)\,dx$$

$$= -\int_{1}^{3} f(x)\,dx - \int_{-2}^{1} f(x)\,dx = -(-6) - 2 = 4$$

29) False

Functions with finitely discontinuities and bounded in $[a,b]$ are also integrable on $[a,b]$. Hence, integrability is necessary, but not sufficient condition for continuity of a function in $[a,b]$.

30) True

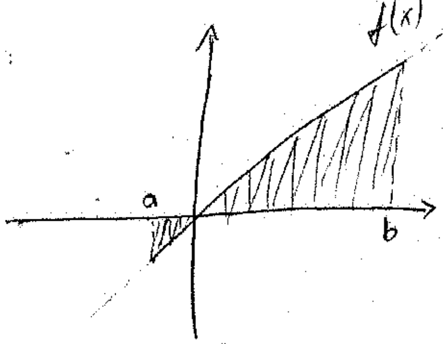
$$\left.\begin{array}{ll} \cos x \geq 0 & \text{for } -1 \leq x \leq 1 \\ \sqrt{1+x^2} \geq 0 & \text{for } -1 \leq x \leq 1 \end{array}\right\} \Rightarrow \frac{\cos x}{\sqrt{1+x^2}} > 0 \text{ for } -1 \leq x \leq 1$$

By theorem 5.5.6 (in 10^{th} ed.)

$$\int_{-1}^{1} \frac{\cos x}{\sqrt{1+x^2}}\,dx > 0$$

31) False

i.e.:



(b >> a)

$$A = \int_{a}^{b} f(x)\,dx > 0$$

However;

$$f(x) \leq 0 \quad \text{for} \quad a \leq x \leq 0$$

32) True

Since $f(x)$ is continuous for every $[a,b]$. Checking $x=0$ point;

$$\lim_{x\to 0^-} 0 = \lim_{x\to 0^+} x^2 = f(0) = 0$$

p320

27 False $|x|$ is continuous so its integral exists.

28 True The average value is attained.

29. $\cancel{\int_a^b f(x)\,dx} =$

Given $F'(x) = f(x)$ & $G'(x) = g(x)$.

then $\int_a^b f(x)\,dx = \int_a^b g(x)\,dx$

$\Rightarrow F(b) - F(a) = G(b) - G(a)$

$\Rightarrow F(b) + G(a) = G(b) + F(a)$

Conversely if $F(b) + G(a) = G(b) + F(a)$

then $F(b) - F(a) = G(b) - G(a)$

$\Rightarrow \int_a^b f(x)\,dx = \int_a^b g(x)\,dx.$

– Don't forget the converse

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By definition.

$\int_0^0 f(t)\,dt = 0$ so $F(0) = 0$

p 336.

$$V(t) = 275{,}000 \sqrt{\frac{20}{t+20}}$$

Leave 275,000 out.

$$\frac{1}{10}\int_0^{10} \sqrt{\frac{20}{t+20}}\, dt$$

$$u = t+20. \qquad \frac{du}{dt} = 1$$

$$\frac{1}{10}\int_{20}^{30} \sqrt{20} \cdot \frac{1}{\sqrt{u}} \cdot du$$

$$\frac{1}{10} \cdot \sqrt{20} \quad \frac{u^{1/2}}{1/2}\Bigg|_{20}^{30}$$

$$= \frac{\sqrt{20}}{10} \cdot 2 \left(\sqrt{30} - \sqrt{20}\right)$$

$$= \frac{1}{5}\left(\sqrt{600} - 20\right)$$

$$= \frac{1}{5}\left(10\sqrt{6} - 20\right)$$

$$= \frac{1}{5}\left(24.5 - 20\right) = 0.90.$$

$$275{,}000 \times .90 = 247{,}500$$

#28.

Note this asks about the average mass not the average rate of growth.

$$r(t) = kt$$

$$\text{mass} : m(t) = \tfrac{1}{2}kt^2 + C$$

$$\text{average} = \frac{1}{26}\int_{26}^{52} \frac{kt^2}{2}\,dt + C$$

$$= \frac{1}{26}\,\frac{kt^3}{6}\Big|_{26}^{52}$$

$$= 788.66k + C.$$

Want

$$\tfrac{1}{2}kt_0^2 + \cancel{C} = 788.66k + \cancel{C}$$

$$t_0 = 39.74.$$