Therefore $$\int_0^1 \frac{dx}{1+x^2} = \frac{0.2}{2}(7.8373) = 0.784 \text{ (approximately).}$$

The exact value is obtained from

$$\int_0^1 \frac{dx}{1+x^2} = \arctan x\Big]_0^1 = \frac{\pi}{4} = 0.7854^-.$$

Before introducing the second method, we establish the *Prismoidal Formula.*

Theorem 4 (Prismoidal Formula). *If $f(x)$ is a polynomial of degree three or less, then*

$$\int_a^{a+2h} f(x)\,dx = \frac{h}{3}[f(a) + 4f(a+h) + f(a+2h)]$$

exactly.

Proof. Let $f(x)$ be any polynomial of degree 3 or less, say

$$f(x) = a + bx + cx^2 + dx^3.$$

When we make the substitution $x = a + h + u$, we have

$$f(a+h+u) = g(u),$$

where $g(u)$ is still a polynomial of the same degree as f. We write

$$g(u) = A + Bu + Cu^2 + Du^3.$$

Then

$$\int_a^{a+2h} f(x)\,dx = \int_{-h}^{h} g(u)\,du = Au + \frac{Bu^2}{2} + \frac{Cu^3}{3} + \frac{Du^4}{4}\Big]_{-h}^{h}$$

$$= 2Ah + \frac{2Ch^3}{3}.$$

From the expression for $g(u)$, we have

$$g(0) = A, \qquad g(h) + g(-h) = 2A + 2Ch^2,$$

and therefore

$$C = \frac{g(h) + g(-h) - 2g(0)}{2h^2}.$$

We conclude that

$$\int_a^{a+2h} f(x)\,dx = 2g(0)h + \frac{h}{3}[g(h) + g(-h) - 2g(0)].$$

We now have $g(0) = f(a+h)$, $g(h) = f(a+2h)$, $g(-h) = f(a)$ and, substituting these values in the last equation, we obtain the Prismoidal Formula.