MA1126 Ass 1. Solutions

p15 #8 (a) $h \wedge w \wedge \sim s$

(b) $h \wedge \sim w \wedge s$

(c) $\sim h \wedge \sim w \wedge \sim s$.

#13 $(p \wedge q) \vee \sim(p \vee q)$.

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \vee \sim(p \vee q)$
T	T	T	T	F	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	T

#45 $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$.

~~$(\sim p \wedge q) \vee \sim(p \vee q)$ by De Morgans~~

$= (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ by De Morgans

$\equiv \sim p \wedge (q \vee \sim q)$ by Distributive

$\equiv \sim p \wedge t$ by $q \vee \sim q \equiv t$.

$= \sim p$ by Identity Law.

48

(a)

p	q	$p \oplus q$	$p \oplus p$	$(p \oplus p) \oplus p$
T	T	F	F	T
T	F	T	F	T
F	T	T	F	F
F	F	F	F	F

So. $p \oplus p \equiv c$.

$(p \oplus p) \oplus p \equiv p$

(b)

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r$	$q \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	T	F	T
T	T	F	F	F	T	F
T	F	T	T	F	T	F
T	F	F	T	T	F	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F
				*	=	*

Yes!

(c)

p	q	r	$p \oplus q$	$(p \oplus q) \wedge r$	$p \wedge r$	$q \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	T	F
T	T	F	F	F	F	F	F
T	F	T	T	F	T	F	T
T	F	F	T	T	F	F	F
F	T	T	T	F	F	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	F	F	F
F	F	F	F	F	F	F	F
				*	$\neq$		*

p27 #14

(a)

p	q	r	$q \vee r$	$p \to q \vee r$	$p \wedge \sim q$	$p \wedge \sim q \to r$	$p \wedge \sim r$	$p \wedge \sim r \to q$
T	T	T	T	T	F	T	F	T
T	T	F	T	T	F	T	T	T
T	F	T	T	T	T	T	F	T
T	F	F	F	F	T	F	T	F
F	T	T	T	T	F	T	F	T
F	T	F	T	T	F	T	F	T
F	F	T	T	T	F	T	F	T
F	F	F	F	T	F	T	F	T

$* = * = *$

(b) n prime $\Rightarrow$ n odd or n = 2

$p \Rightarrow q \vee r$.

$p \wedge \sim q \Rightarrow r$

n prime and not odd $\Rightarrow$ n = 2

$p \wedge \sim r \Rightarrow q$

n prime and not = 2 $\Rightarrow$ n is prime

#16 (a) ~~There are rectangles that are not squares.~~

~~(a)~~
(a) Some P is a square and P is not a rectangle

(b) Some Thanksgivings ~~days~~ are not follow by Friday.

(c) Some rationals do not have a repeating dec exp

(d) Some primes are not (odd or 2)

= not odd and not 2.

24 Sq ⇒ Mean

Mean ⇒ Sq not 2.

40 (a) body ⇒ > 250° F

(a) No better reach 500°.

(b) T. Contra posit

(c) ~~T~~ F. bone ⇒ > 240° F

(d) F.

p39 #6

p	q	$p \to q$	$q \to p$	$p \vee q$	
T	T	T	T	T	critical row ✓
T	F	F	T		
F	T	T	F		
F	F	T	T	F	critical row ✓

∴ Invalid argument.

#9

p	q	r	$p \to q$	$p \to r$	$q \wedge r$	$p \to q \wedge r$	
T	T	T	T	T	T	T	✓
T	T	F	T	F	F		
T	F	T	F	T	F		
T	F	F	F	F	F		
F	T	T	T	T	T	T	✓
F	T	F	T	T	F	T	✓
F	F	T	T	T	F	T	✓
F	F	F	T	T	F	T.	✓

∴ Valid argument.

#39 Socks T ⇒ Muscles False ⇒ Lefty
⇒ Fats False ⇒ Muscles
contradiction.
∴ Socks False ⇒ not L
⇒ Muscles True.
⇒ Fats False ⇒ Muscles
+ Lefty False is shown
∴ Muscles did it.

#41.

Assignment 2

Q1 Consider the table

X	Y	Z	output
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

Write the disjunctive normal form for this table.

Write out the circuit for this form

Next simplify the normal form as much as you can and write out the circuit for the new form.

Q2. Design a logic circuit that inputs the values of three variables x, y, z and outputs a 1 if and only if $x = y$ and $x \neq z$.

Q3. Use Math. Ind. to prove that if X has n elements then $P(X)$ has 2^n elements.

Q4 Let $(x_1, y_1) \sim (x_2, y_2)$ mean $y_1 = y_2$. Show this is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.

Q5. On $\mathbb{N} = \{1, 2, 3, 4, \cdots\} \times \mathbb{N}$

define $(n_1, m_1) \sim (n_2, m_2)$ means $n_1 + m_2 = n_2 + m_1$

Show $\sim$ is an equivalence relation.

Q6. On $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$, define

$(a_1, b_1) \sim (a_2, b_2)$ means $a_1 b_2 = a_2 b_1$

Show $\sim$ is an equivalence relation.

MA 1126 Assignment 2 Solutions.

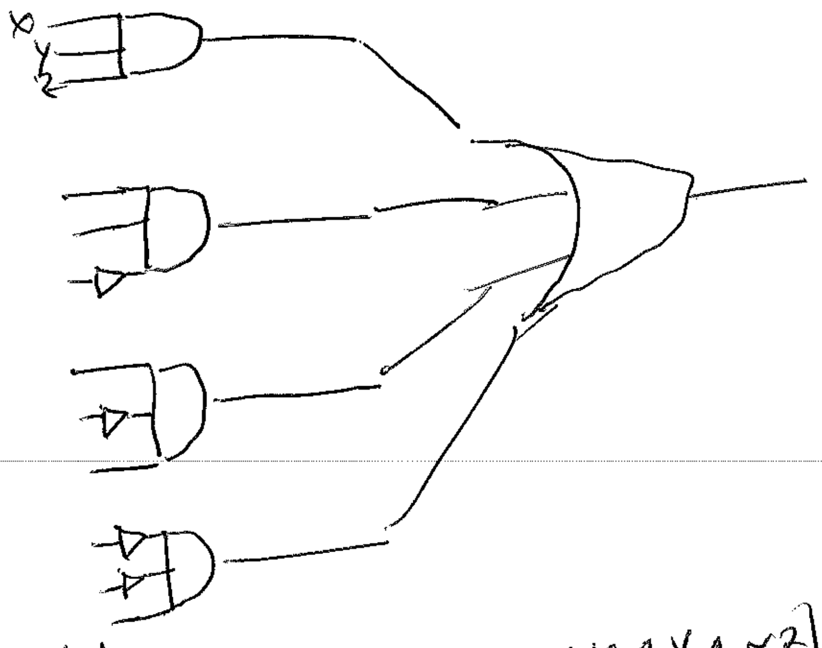
1.

x	y	z	output
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

The disjunctive normal form is

$(x \wedge y \wedge z) \vee (x \wedge y \wedge \sim z) \vee (x \wedge \sim y \wedge z) \vee (\sim x \wedge \sim y \wedge z)$.

Circuit



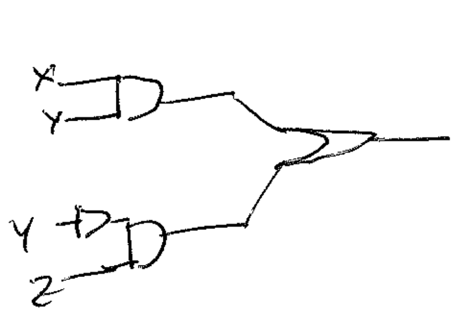
1

Simplifying $(x \wedge y \wedge z \vee x \wedge y \wedge \sim z) \equiv x \wedge y \wedge (z \vee \sim z)$
$\equiv x \wedge y$.

$(x \wedge \sim y \wedge z) \vee (\sim x \wedge \sim y \wedge z) \equiv (x \vee \sim x) \wedge (\sim y \wedge z)$
$\equiv \sim y \wedge z$.

New circuit

1



Much Better.

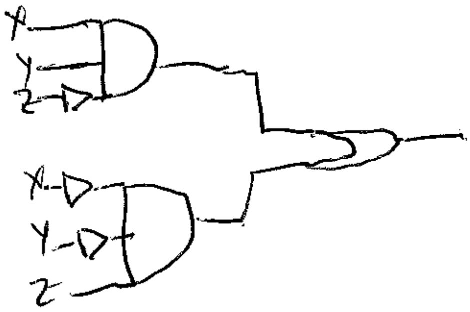
2.

X	Y	Z	output	
1	1	1	0	
1	1	0	1	$(X \wedge Y \wedge \sim Z)$
1	0	1	0	
1	0	0	0	
0	1	1	0	
0	1	0	0	
0	0	1	1	$(\sim X \wedge \sim Y \wedge Z)$.
0	0	0	0	

Disjunctive Normal Form

$(X \wedge Y \wedge \sim Z) \vee (\sim X \wedge \sim Y \wedge Z)$

Circuit



1

3. X has n elements ⇒ $\mathcal{P}(X)$ has 2^n elements

Let $\# X = 1$ $X = \{1\}$. $\mathcal{P}(X) = \{X, \emptyset\}$
$= 2^1$ elements

Suppose ~~Pn~~ $P(n)$ is X has n elements
⇒ $\mathcal{P}(X)$ has 2^n elts.

1 $P(1)$ is True.
Suppose $P(k)$ is True.
if X has $k+1$ elts. Take one away,
then we have 2^k subsets. Now
add this extra element to each of
the 2^k subsets. This gives us
2^k new subsets + 2^k old
$= 2^{k+1}$

4. On $\mathbb{R} \times \mathbb{R}$ $(x_1, y_1) \sim (x_2, y_2)$ means $y_1 = y_2$

1 Reflexive $(x_1, y_1) \sim (x_1, y_1)$ since $y_1 = y_1$

2 Symmetric $(x_1, y_1) \sim (x_2, y_2)$ means $y_1 = y_2$

$\Rightarrow y_2 = y_1 \Rightarrow (x_2, y_2) \sim (x_1, y_1)$

(1) 3. Transitive $(x_1, y_1) \sim (x_2, y_2)$ mean $y_1 = y_2$

$(x_2, y_2) \sim (x_3, y_3)$ means $y_2 = y_3$

$\Rightarrow y_1 = y_3 \Rightarrow (x_1, y_1) \sim (x_3, y_3)$

5. on $\mathbb{N} \times \mathbb{N}$. $(n_1, m_1) \sim (n_2, m_2)$ means $n_1 + m_2 = m_1 + n_2$

1. Reflexive $(n, m) \sim (n, m)$ since $n + m = m + n$

2. Symmetric $(n_1, m_1) \sim (n_2, m_2) \Rightarrow n_1 + m_2 = m_1 + n_2$

$\Rightarrow m_1 + n_2 = n_1 + m_2 \Rightarrow n_2 + m_1 = m_2 + n_1$

(1) $\Rightarrow (n_2, m_2) \sim (n_1, m_1)$

3. Transitive $(n_1, m_1) \sim (n_2, m_2) \Rightarrow n_1 + m_2 = m_1 + n_2$

$(n_2, m_2) \sim (n_3, m_3) \Rightarrow n_2 + m_3 = m_2 + n_3$

$\Rightarrow n_1 + m_2 + n_2 + m_3 = m_1 + n_2 + m_2 + n_3$

$\Rightarrow n_1 + m_3 = m_1 + n_3$.

$\Rightarrow (n_1, m_1) \sim (n_3, m_3)$

6 $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ $(a_1, b_1) \sim (a_2, b_2)$ mean $a_1 b_2 = a_2 b_1$

R. $(a_1, b_1) \sim (a_1, b_1)$ since $a_1 b_1 = a_1 b_1$ ✓

S. $(a_1, b_1) \sim (a_2, b_2) \Rightarrow a_1 b_2 = a_2 b_1$

$\Rightarrow a_2 b_1 = a_1 b_2$

(2) $\Rightarrow (a_2, b_2) \sim (a_1, b_1)$

T. $(a_1, b_1) \sim (a_2, b_2) \Rightarrow a_1 b_2 = b_1 a_2$
$(a_2, b_2) \sim (a_3, b_3) \Rightarrow a_2 b_3 = b_2 a_3$ $\Big\}$ $a_1 b_2 a_2 b_3 = b_1 a_2 b_2 a_3$

$\Rightarrow a_1 b_3 =$ P.T.O.

MA 1126 Assignment 3

Q 1, 2, 3 are find the equivalence classes for 4, 5, 6 on assignment 2. Give a nice "geometric" description.

4. If $f: X \to Y$ is onto is the induced $f: P(X) \to P(Y)$ onto?

5. Prove $f^{-1}(\bigcup_i A_i) = \bigcup_i f^{-1}(A_i)$.

6. Prove that $f: X \to Y$ is 1-1 if and only if $f(A \cap B) = f(A) \cap f(B)$ all $A, B \subset X$.

7. Prove that f is 1-1 and onto if and only if $f(A^c) = (f(A))^c$ $\forall A \subset X$.

8. Prove that f is 1-1 if and only if $f^{-1} \circ f(A) = A$ $\forall A \subset X$.

Solutions Assignment 3 MA1126.

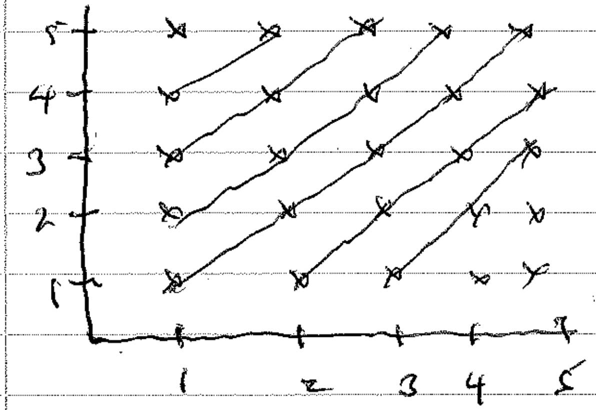
Q 1 on $\mathbb{R}\times\mathbb{R}$ $(x_1, y_1) \sim (x_2, y_2)$ means $y_1 = y_2$.

The equivalence classes are the lines $y =$ constant.

1

A "nice" set of representatives is the y-axis, or any line $x =$ const.

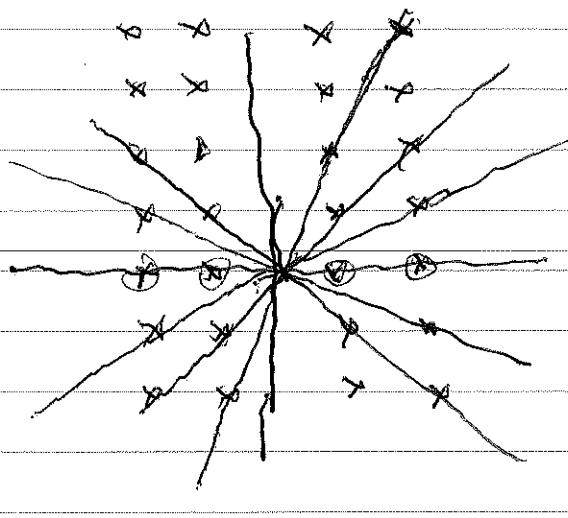
Q2. on $\mathbb{N}\times\mathbb{N}$ $(n_1, m_1) \sim (n_2, m_2)$ means $n_1 + m_2 = m_1 + n_2$



The equivalence classes are the ~~lines~~ points on the lines

2

Q3 on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ $(a_1, b_1) \sim (a_2, b_2)$ means $a_1 b_2 = b_1 a_2$



The equivalence classes are ~~the~~ points on the lines

2

#16 (a) ~~There are rectangles that are not squares~~

~~(b)~~

(a) Some P is a square and P is not a rectangle

(b) Some Thanksgivings ~~Holidays~~ are not follow by Friday.

(4) (c) Some rationals do not have a repeat dec exp

(d) Some primes are not (odd or 2)

= not odd and not 2

24 Say ⇒ Mean

(1) Mean ⇒ Say not =

40 (a) body ⇒ > 250°F

(a) No baker make 500°

(b) T. Contrapos

(4) (c) ~~T~~ bac ⇒ > 240°F

(d) F.

Assignment 4 MA1126

Q1, 2, 3 Do 6, 7, 8 from Ass 2.

Be very clear in each step.

Remember P iff Q means $P \Rightarrow Q$ & $Q \Rightarrow P$.

Q4.

$$\begin{tikzcd} \mathbb{N} \times \mathbb{N} \arrow[r, "\times"] \arrow[d, "p \times p"'] & \mathbb{N} \arrow[d, "p"] \\ \mathbb{Z} \times \mathbb{Z} \arrow[r, "\times"] & \mathbb{Z} \end{tikzcd}$$

Show the diagram commutes.

Q5 Do 27 in supp. problems p44

Q6 Do 28 " " " p45

Hint look at worked solutions.

Assignment 4 MA 1126

Q4.

$$\begin{tikzcd} \mathbb{N} \times \mathbb{N} \arrow[r, "\text{mult}"] \arrow[d, "\varphi \times \varphi"'] & \mathbb{N} \arrow[d, "\varphi"] \\ \mathbb{Z} \times \mathbb{Z} \arrow[r, "\text{mult}"] & \mathbb{Z} \end{tikzcd}$$

Show the diagram commutes

(1)

$$\begin{tikzcd} (n, m) \arrow[r, mapsto] \arrow[d] & nm \arrow[d] \\ {[(1+n, 1)], [(1+m, 1)]} \arrow[r] & {[(1+nm, 1)]} \end{tikzcd}$$

Note $[(a,b)] \cdot [(c,d)] \to [(ac+bd,\ bc+ad)]$.

$$\to [((1+n)(1+m)+1,\ (1+m)+(1+n))]$$

$$= [(2+nm+n+m,\ 2+n+m)]$$

$$= [(1+nm,\ 1)].$$

Q5. The set of points in $\mathbb{R}^2$ with rational co-ords is denumerable.

This set is $\mathbb{Q} \times \mathbb{Q}$ and we

(1) know $\mathbb{Q} \sim \mathbb{N}$, so $\mathbb{Q}\times\mathbb{Q} \sim \mathbb{N}\times\mathbb{N}$ which we know to be denumerable

Q6. Prove that the set of transcendental numbers is non denumerable

From the earlier questions we know the set of algebraic #'s is denumerable, so if all the other

(1) numbers is denumerable, then since denumerable $\cup$ denumerable = denumerable then $\mathbb{R}$ is denumerable, ~~hence~~ And we know its not, so the transcendentals are <u>not</u> denumerable.

6 $f: X \to Y$ is 1-1 if and only if
$\cancel{f(A \cap f(B))}$ $f(A \cap B) = f(A) \cap f(B) \quad \forall A, B \subset X$.

if f is not 1-1. $\exists\, x_1 \neq x_2$ s.t. $f(x_1) = f(x_2)$

Let $A = \{x_1\}$ $\quad B = \{x_2\}$.

(1) then $A \cap B = \phi$.
$f(A \cap B) = \phi$ but $f(A) \cap f(B) = \{f(x_1)\}$

If Let $f(A \cap B) = f(A) \cap f(B) \quad \forall A, B$.

Let $f(x_1) = f(x_2)$
then $A = \{x_1\}$ $B = \{x_2\}$

(1) $f(A \cap B) = f(A) \cap f(B) = \{f(x_1)\}$
$\therefore A \cap B \neq \phi$
$\therefore x_1 = x_2$
$\therefore f$ is 1-1.

7. f is 1-1 and onto iff $f(A^c) = f(A)^c \quad \forall A \subset X$.

Let $f(x_1) = f(x_2)$
Let $A = \{x_1\}$. $A^c = X \setminus \{x_1\}$

(1) $f(x_1) = f(x_2) \notin f(A^c)$ but $x_2 \in A^c$ $\therefore f(x_2) \in f(A^c)$
if $x_2 \neq x_1$

~~But~~ Hence $x_2 = x_1$.

$\therefore f$ is 1-1

~~Let $\exists \in Y$, $\ni$ $y \neq f(x)$ any x.~~

~~then~~

(1) Given $f(A^c) = f(A)^c$. Let $A = \phi$.

then $f(X) = \phi^c = Y$. so f is onto.

Conversely If f is 1-1 and onto

Let $y \in f(A^c)$ then $\exists\, x_1 \in A^c$, $y = f(x_1)$

But 1-1 $\Rightarrow$ $\nexists\, x_2 \in A$ s.t. $y = f(x_2)$

$\therefore$ $y \notin f(A)$ $\therefore$ $y \in f(A)^c$

$\therefore$ $f(A^c) \subset f(A)^c$.

Let $y \in f(A)^c$ then $y \notin f(A)$.

(2) onto $\Rightarrow$ $y = f(x)$ some x

$\therefore$ $x \notin A$ $\therefore$ $x \in A^c$

$\therefore$ $y \in f(A^c)$

$\therefore$ $f(A)^c \subset f(A^c)$

$\therefore$ $f(A^c) = f(A)^c$.

8 f is 1-1 if and only if

$$f^{-1} \circ f(A) = A \quad \forall A \subset X.$$

Given 1-1 Let $x \in A$.

Then $f(x) \in f(A)$

$\Rightarrow f^{-1}(f(A)) \ni x$.

$\therefore A \subset f^{-1}(f(A))$ always.

(1) Let $x \in f^{-1} \circ f(A)$

then $f(x) \in f(A)$.

i.e. $\exists x_1 \in A$ s.t. $f(x_1) = f(x)$

Then 1-1 $\Rightarrow x_1 = x \in A$.

$\therefore f^{-1} \circ f(A) \subset A$

$\therefore f^{-1} \circ f(A) = A$.

Conversely

if $f^{-1} \circ f(A) = A \quad \forall A$.

Let $f(x_1) = f(x_2)$

Let $A = \{x_1\}$.

$f(x_1) \in f(A)$

(1) $\therefore f(x_2) \in f(A)$

$\therefore x_2 \in f^{-1} \circ f(A)$

$\therefore x_2 \in A \quad \therefore x_2 = x_1$

$\therefore f$ is 1-1.

Assignment 5 MA1126

Q 1. Use Zorn's Lemma to prove any A, B sets $A < B$, $A \sim B$ or $B < A$.

We want a function $f : A \to B$ that is 1-1. Suppose we just have $f : C \to B$ where $C \subset A$, f 1-1.

Let X = the collection of these pairs C, f.

Given (C_1, f_1) and (C_2, f_2) define

$(C_1, f_1) \le (C_2, f_2)$ means

$C_1 \subset C_2$ and $f_2 = f_1$ on C_1

so f_2 is an extension of f_1.

Check this is a p.o.

Suppose $\{C_\alpha, f_\alpha\}$ is a totally ordered subset. We want an upper bound in X. Let $C = \bigcup C_\alpha$ and $f(x) = f_{\alpha_0}(x)$ if $x \in C_{\alpha_0}$. This is well defined and 1-1 (Check)

So by Zorn's Lemma $\exists$ a maximal element (D, g)

If $D = A$, and $g(D) = B$ then $A \sim B$.

If $D = A$ and $g(D) \neq B$ then $A < B$.

If $D \neq A$ and $g(D) = B$ then $B < A$

$g: D \to B$ is 1-1 + onto so $g^{-1}: B \to A$.

If $D \neq A$ and $f(D) \neq B$.

Let $a \in A \setminus D$ and $b \in B \setminus f(D)$

Let $h: D \cup \{a\} \to f(D) \cup \{b\}$

by $d \in D$, $d \to g(d)$ and $a \to b$

then $(D \cup \{a\}, h) > (D, g)$

$\Rightarrow \Leftarrow$.

So one of the top three statements is true.

#40. Let $x_1, x_2, \ldots, x_n$ be in $X, \leq$.

If x_1 is maximal we are done. If not $\exists\, x_i > x_1$.

If x_i is maximal, we are done, if not $\exists\, x_{i_j} > x_i$

If x_{i_j} is maximal, etc.

This terminates after at most n steps.

#43 But $A = \bigcup_i B_i$ is not in $\mathcal{A}$ since it might not be finite.

#46. Given $(X, \leq)$. Define $\leq_1$ on X by $x_1 \leq_1 x_2$ means $x_2 \leq x_1$.

Then every totally ordered subset of $\leq_1$ is a totally ordered subset of $\leq$ and vice versa.

~~Let~~ Take any totally ordered subset of $(X, \leq_1)$, then it is totally ordered in $\leq$ and has a lower bound which is an upper bound of $\leq_1$. So by Zorn we have a max. elt for $\leq_1$ which is a min. elt. for $\leq$.

Assignment 6 MA 1126

1. Prove that a is an interior point of $A \subset \mathbb{R}$ $\iff \exists\, \epsilon > 0$ s.t. $N(\epsilon, a) \subset A$.

Recall $N(\epsilon, a) = \{ b : |a - b| < \epsilon \}$.

2. Prove that a ~~[illegible]~~ is an acc. point of $A \iff \exists\, x_n \in A,\ n = 1, 2, 3, \ldots$ $x_n \neq a$ such that $x_n \to a$.

Do # 38-44 below

Supp...

OPEN SETS, CLOSED SETS, ACCUMULATION POINTS

38. Prove: If A is a finite subset of $\mathbf{R}$, then the derived set A' of A is empty, i.e. $A' = \emptyset$.

39. Prove: Every finite subset of $\mathbf{R}$ is closed.

40. Prove: If $A \subset B$, then $A' \subset B'$.

41. Prove: A subset B of $\mathbf{R}^2$ is closed if and only if $d(p, B) = 0$ implies $p \in B$, where $d(p,$ inf $\{d(p, q) : q \in B\}$.

42. Prove: $A \cup A'$ is closed for any set A.

43. Prove: $A \cup A'$ is the smallest closed set containing A, i.e. if F is closed and $A \subset F \subset A \cup A'$ $F = A \cup A'$.

44. Prove: The set of interior points of any set A, written int(A), is an open set.

45. Prove: The set of interior points of A is the largest open set contained in A, i.e. if G is open int$(A) \subset G \subset A$, then int$(A) = G$.

46. Prove: The only subsets of $\mathbf{R}$ which are both open and closed are $\emptyset$ and $\mathbf{R}$.

SEQUENCES

47. Prove: If the sequence $\langle a_n \rangle$ converges to $b \in \mathbf{R}$, then the sequence $\langle |a_n - b| \rangle$ converges to 0.

48. Prove: If the sequence $\langle a_n \rangle$ converges to 0, and the sequence $\langle b_n \rangle$ is bounded, then the seq $\langle a_n b_n \rangle$ also converges to 0.

49. Prove: If $a_n \to a$ and $b_n \to b$, then the sequence $\langle a_n + b_n \rangle$ converges to $a + b$.

50. Prove: If $a_n \to a$ and $b_n \to b$, then the sequence $\langle a_n b_n \rangle$ converges to ab.

Assignment 6 MA 1126.

Q1. If a is an interior point of A then $\exists$ I_a an open interval containing ~~$\nexists$~~ a contained in A.

If $I_a = (b, c)$, let $\epsilon = \min(a-b, c-a)$. then $N(\epsilon, a) \subset I_a \subset A$.

$N(\epsilon, a) = (a-\epsilon, a+\epsilon)$.

Conversely if $\exists$ $N(\epsilon, a) \subset A$, then this is an I_a.

Q2. Suppose a is an acc. point of A. then by defn. every open set containing a contains a point of A $\neq a$.

So for $N(\frac{1}{n}, a)$ choose $x_n \in A$ $x_n \neq a$

then $x_n \to a$

Conversely if $x_n \in A$, $x_n \neq a \to A$.

Let G be open, $a \in G$. then $\exists$ $N(\epsilon, a) \subset G$. ~~If $\frac{1}{n} < \epsilon$, then $x_n \in N(\epsilon, a)$~~

given ϵ $\exists$ N s.t. $n \geq N$

$\Rightarrow |x_n - a| < \epsilon$

$\Rightarrow x_n \in G$. $\therefore G \cap (A \setminus \{a\}) \neq \phi$.

$\therefore$ a is an acc. point.

#38. If A is finite, A cannot have a sequence $x_n \neq a$, $x_n \to a \in A$, $x_n \in A$.

#39. Hence a finite ~~cor~~ set contains all of its acc. points ~~so~~ is closed.

#40. Given $A \subset B$, let $a \in A'$, ~~then~~ then ~~$a \in A$~~ ~~or~~ a is an acc. point

so $\exists x_n \in A$, $x_n \neq a$, $x_n \to a$

$\therefore \exists x_n \in B$, $x_n \neq a$ $x_n \to a$

$\therefore a$ is an acc. point of B.

#41. $d(p, B) = 0 \iff \inf_{b \in B} |p - b| = 0$.

$\iff p \in B$ or $\exists b_n \to p$, $b_n \in B$.

$\iff p \in B' \cup B$.

B closed $\iff B' \subset B$

$\therefore$ $p \in B \Rightarrow d(p, B) = 0$.

& $d(p, B) = 0$ ~~$\Rightarrow p$~~ $\Rightarrow p \in B$

$\Rightarrow B' \subset B \Rightarrow$ closed.

then if $i > k$ and $n > N$.

$$|x_{i_N, N} - a| \leq |x_{i_N, N} - x_N| + |x_N - a| < \epsilon/2 + \epsilon/2 = \epsilon.$$

43. F closed $A \subset F \subset A \cup A' \Rightarrow F = A \cup A'$.

F ~~closed ⇒ all its~~ acc. points are ~~in F ⇒~~

F closed $\Rightarrow F = F'$.

$A \subset F \Rightarrow A' \subset F' \therefore A' \subset F$

$\therefore F = A \cup A'$.

44. Let A^0 = set of interior points of A.

Let $a \in A^0$, then $\exists\, N(\epsilon, a) \subset A$

$[a-\epsilon \quad a+\epsilon]$ For any point in this interval is also an interior point of A, so $N(\epsilon, a) \subset A^0$

$\therefore A^0$ is open.

Assignment 6 MA 1126

1. Prove that a is an interior point of $A \subset \mathbb{R}$ $\Leftrightarrow \exists\ \epsilon > 0$ s.t. $N(\epsilon, a) \subset A$.

Recall $N(\epsilon, a) = \{ b : |a - b| < \epsilon \}$.

2. Prove that a ~~[illegible]~~ is an acc. point of $A \Leftrightarrow \exists\ x_n \in A,\ n = 1, 2, 3, \ldots$ $x_n \neq a$ such that $x_n \to a$.

Do # 38 - 44 below

Why symmetric?

Prove that $\{x_n\}$ is closed [illegible]

A set is not open / closed

$A \subset F$ & $A' \subset F' \subset F$ since closed

$\therefore A \cup A' \subset F$

x not a bd. pt of A

$\Rightarrow \exists$

$A' = \phi \Rightarrow$ closed

Assignments 7+8 MA1126

Due Wed 21st

1. Properties of $\bar{A} = A \cup A'$.

(i) $\bar{A}$ is closed

(ii) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$

(iii) $A \subset \bar{A}$

(iv) A is closed $\Leftrightarrow A = \bar{A}$

(v) $\bar{A}$ is smallest closed superset of A.

(vi) $\bar{A} = \bigcap_{H \text{ closed} \supset A} H$

(vii) A is closed $\Leftrightarrow \forall x_n \in A$, $x_n \to x \Rightarrow x \in A$.

(viii) $\bar{\bar{A}} = \bar{A}$

2 Properties of $A^\circ = \text{Int} A$.

Do (i) – (viii) of corresponding properties

(vii) becomes A is open $\Leftrightarrow x_n \to x \in A \Rightarrow \exists N$ s.t. $n \geq N \Rightarrow x_n \in A$.

3 Prove (i) $(A^o)^c = \overline{A^c}$

(ii) $(\bar{A})^c = (A^c)^o$.

4 If $\delta A = A' \cap (A^c)'$

show $\delta A = (A^o \cup (A^c)^o)^c$

5 (i) Prove l.u.b $A \in A$ if A is closed.

(ii) Prove l.u.b $A \notin A$ if A is open.

6 Prove $x_n \to x \Rightarrow$ every subsequence $x_{n_k} \to x$.

Assignments 7 + 8 MA1126

Q1 Properties of $\bar{A} = A \cup A'$.

<u>Q 1</u> (i) $\bar{A}$ is closed

Let $x_n \to x$, $x_n \in \bar{A}$, $x_n \neq x$.

if inf many $x_n \in A$, then $x \in \bar{A}$ ✓

if not $x_n \in \bar{A}$ $\exists\, x_{n_i} \to x_n$. $x_{n_i} \in A$.

$\forall i, \exists\, x_{N_i, i}$ s.t. $n > N_i$
$\Rightarrow |x_{n,i} - x_n| < \frac{1}{2^i}$

then we can show $\{x_{N_i, i}\} \to x$

$\therefore x \in \bar{A}$,

$\therefore \bar{A}$ is closed

(ii) $A \subset B \Rightarrow$ any acc. pt. of A is an acc. point of B
$\Rightarrow \bar{A} \subset \bar{B}$.

(iii) $A \subset \bar{A}$ by def.

(iv) A is closed $\Leftrightarrow A = \bar{A}$

$\Leftarrow$ follows from (i).

$\Rightarrow$ follows from defn $\bar{A} = A \cup A'$

(v) Let $A \subset H \subset \bar{A}$ $\quad \bar{A}$ closed

by (ii) $\bar{A} \subset \bar{H} \subset \bar{A}$ $\therefore$ $\bar{A} = \bar{H}$.

(vi) H closed $\supset A$ $\quad \cap H =$ closed

and $\bar{A} =$ one H

so $\cap H \subset \bar{A}$ $\therefore$ $\bar{A} = \cap H$.

(vii) If A closed + $x_n \in A$, $x_n \to x$,

if inf many $x_n \neq x$, then $x \in A'$.

if inf many $x_n = x$, then $x \in A$

$\therefore$ $x \in \bar{A} = A$.

(viii) $\bar{\bar{A}} = \bar{A}$.

$(\bar{\bar{A}})$ = smallest closed set containing

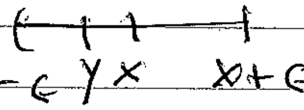
$\bar{A}$, $\bar{A}$ closed $\Rightarrow$ $\bar{\bar{A}} = \bar{A}$.

Q2 Properties of A°.

(i) A° is open.

Let $x \in A^\circ$, then $\exists\, N(\epsilon, x) \subset A$.

and if $y \in N(\epsilon, x)$, then $\exists\, N(\delta, y) \subset N(\epsilon, x) \subset A$



$\therefore y \in A^\circ$.

$\therefore N(\epsilon, x) \subset A^\circ \therefore x$ is an interior point of A°.

(ii) $A \subset B \Rightarrow A^\circ \subset B^\circ$.

$x \in A^\circ \Rightarrow \exists\, N(\epsilon, x) \subset A$

$\therefore N(\epsilon, x) \subset B$

$\therefore x \in B^\circ$.

(iii) $A^\circ \subset A$ by defn

(iv) A is open $\Leftrightarrow A = A^\circ$.

$\Leftarrow$ is from (i).

$\Rightarrow$ is from defn

(v) A° is the largest open subset of A.

Let $A^\circ \subset G \subset A$ G open.

$G \subset A \Rightarrow G^\circ = G \subset A^\circ$ by (ii)

$\therefore G = A^\circ$

(vi) $A^\circ = \bigcup_{G \text{ open} \subset A} G$.

$\bigcup G$ is open and contains A°

$\therefore =$ by (v).

(vii) A is open $\Leftrightarrow$ $x_n \to x \in A$

$\Rightarrow \exists N$ s.t. $n \geq N$

$\Rightarrow x_n \in A$.

$\Rightarrow$ if A is open $x \in A$

$\exists N(\epsilon, x) \subset A$, $x_n \to x \Rightarrow$

$\exists N$ s.t. $n \geq N \Rightarrow |x_n - x| < \epsilon$

so $x_n \in A$.

$\Leftarrow$ if A is not open $\exists x$ s.t.

$\forall \epsilon$, $N(x, \epsilon) \not\subset A$ i.e. $N(x, \epsilon) \cap A^c \neq \phi$.

$\exists x_n$ s.t. $N(x, 1/n) \cap A^c \ni x_n$.

Then $x_n \to x$, but $\forall n$, $x_n \notin A$.

(viii) $(A^\circ)^\circ = A^\circ$.

A° is open by (i)

$\Rightarrow (A^\circ)^\circ = A^\circ$ by (iv).

3. (i) $(A^\circ)^c = \overline{A^c}$

Let $x \in (A^\circ)^c \Leftrightarrow x \notin A^\circ$

$\Leftrightarrow \forall \epsilon \; N(\epsilon, x) \cap A^c \neq \emptyset$.

$\Leftrightarrow \exists \; x_n \in A^c, \; x_n \to x$

$\Leftrightarrow x \in \overline{A^c}$

(ii) $(\bar{A})^c = (A^c)^\circ$

$x \in (\bar{A})^c \Leftrightarrow x \notin \bar{A}$.

$\Leftrightarrow \nexists \; x_n \in A, \; x_n \to x$.

$\Leftrightarrow \exists \; N(\epsilon, x) \subset A^c$

$\Leftrightarrow x \in (A^c)^\circ$.

4. $\delta A = A' \cap (A^c)'$

Show $\delta A = (A^\circ \cup (A^c)^\circ)^c$.

$x \in A'$ and $(A^c)'$

$\Leftrightarrow \exists \; x_n \in A, \; x_n \neq x, \; x_n \to x$.

$\& \; \exists \; y_n \in A^c, \; y_n \neq x, \; y_n \to x$

$\Leftrightarrow x \notin (A^c)^\circ$

$\& \; x \notin A^\circ$

$\Leftrightarrow x \notin A^\circ \cup (A^c)^\circ$

$\Leftrightarrow x \in (A^\circ \cup (A^c)^\circ)^c$.

5. (i) A closed $\Rightarrow$ l.u.b. $A \in A$.

Let $a =$ l.u.b. A.

$a - 1/n$ is not an upper bound

so $\exists\ x_n \in A,\ x_n > a - 1/n$.

$x_n \to a$ so a is an

acc point, ~~or~~ $x_n = a$.

in either case $a \in A$.

(ii) A open $\Rightarrow$ l.u.b. $A \notin A$.

Let $a =$ l.u.b. a.

if $a \in A$ $\exists\ N(a, \epsilon) \subset A$.

$\therefore\ a + \epsilon/2 \in A$

$\therefore\ a$ not l.u.b. $\Rightarrow \Leftarrow$.

6. Let $x_n \to x$.

given $\{x_{n_i}\}$, given $\epsilon > 0$.

$\exists\ N$ s.t. $n \geq N \Rightarrow |x_n - x| < \epsilon$.

Let $M = n_N \geq N$.

then $n_i \geq M \Rightarrow |x_{n_i} - x| < \epsilon$.