Assignments 7+8 MA1126

Due Wed 21st

1. Properties of $\bar{A} = A \cup A'$.

(i) $\bar{A}$ is closed

(ii) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$

(iii) $A \subset \bar{A}$

(iv) A is closed $\Leftrightarrow A = \bar{A}$

(v) $\bar{A}$ is smallest closed superset of A.

(vi) $\bar{A} = \bigcap\limits_{H \text{ closed} \supset A} H$

(vii) A is closed $\Leftrightarrow \forall x_n \in A,\ x_n \to x \Rightarrow x \in A$.

(viii) $\bar{\bar{A}} = \bar{A}$

2 Properties of $A^\circ = \operatorname{Int} A$.

Do (i) - (viii) of corresponding properties

(vii) becomes A is open $\Leftrightarrow x_n \to x \in A \Rightarrow \exists N$ s.t. $n \geq N \Rightarrow x_n \in A$.

3 Prove (i) $(A^{o})^{c} = \overline{A^{c}}$

(ii) $(\overline{A})^{c} = (A^{c})^{o}$.

4 If $\delta A = A' \cap (A^{c})'$

show $\delta A = (A^{o} \cup (A^{c})^{o})^{c}$

5. (i) Prove l.u.b $A \in A$ if A is closed.

(ii) Prove l.u.b $A \notin A$ if A is open.

6 Prove $x_n \to x \Rightarrow$ every subsequence $x_{n_k} \to x$.