Assignments 7+8

Due Wed 21st.

1. Properties of \( \bar{A} = A \cup A' \).

   (i) \( \bar{A} \) is closed

   (ii) \( A \subseteq B \Rightarrow \bar{A} \subseteq \bar{B} \)

   (iii) \( A \subseteq \bar{A} \)

   (iv) \( A \) is closed \( \Leftrightarrow \) \( A = \bar{A} \)

   (v) \( \bar{A} \) is smallest closed superset of \( A \).

   (vi) \( \bar{A} = \bigcap H \)  
        \( H \) closed \( \supseteq A \)

   (vii) \( A \) is closed \( \Leftrightarrow \forall x_n \in A, x_n \to x \Rightarrow x \in A \).

   (viii) \( \bar{A} = \bar{\bar{A}} \)

2. Properties of \( A^0 = \text{Int} A \).

   Do (i - viii) of corresponding properties

   (vii) because \( A \) is open \( \Rightarrow \exists x_n \subseteq x \in A \)

   \( \Rightarrow x_n \neq x \in A \)
4. \( \bar{\delta A} = \delta A \cap (A^c)^c \)

Show \( \delta A = (A^c \cup (A^c)^c)^c \)

5. (i) Prove \( \text{d.u.b. } A = A \) if \( A \) is closed.

(ii) Prove \( \text{d.u.b. } A = A \) if \( A \) is open.

6. Prove \( x_n \to x \implies \) every subsequence \( x_{n_k} \to x \)