Assignment 6 MA 1126

1. Prove that a is an interior point of $A \subset \mathbb{R}$ $\Leftrightarrow \exists\, \epsilon > 0$ s.t. $N(\epsilon, a) \subset A$.

Recall $N(\epsilon, a) = \{ b : |a - b| < \epsilon \}$.

2. Prove that a is an acc. point of $A \Leftrightarrow \exists\, x_n \in A,\ n = 1, 2, 3, \ldots,\ x_n \neq a$ such that $x_n \to a$.

Do # 38-44 below

OPEN SETS, CLOSED SETS, ACCUMULATION POINTS

38. Prove: If A is a finite subset of $\mathbf{R}$, then the derived set A' of A is empty, i.e. $A' = \emptyset$.

39. Prove: Every finite subset of $\mathbf{R}$ is closed.

40. Prove: If $A \subset B$, then $A' \subset B'$.

41. Prove: A subset B of $\mathbf{R}^2$ is closed if and only if $d(p, B) = 0$ implies $p \in B$, where $d(p, B) =$ $\inf\{d(p, q) : q \in B\}$.

42. Prove: $A \cup A'$ is closed for any set A.

43. Prove: $A \cup A'$ is the smallest closed set containing A, i.e. if F is closed and $A \subset F \subset A \cup A'$ then $F = A \cup A'$.

44. Prove: The set of interior points of any set A, written $\operatorname{int}(A)$, is an open set.

45. Prove: The set of interior points of A is the largest open set contained in A, i.e. if G is open and $\operatorname{int}(A) \subset G \subset A$, then $\operatorname{int}(A) = G$.

46. Prove: The only subsets of $\mathbf{R}$ which are both open and closed are $\emptyset$ and $\mathbf{R}$.

SEQUENCES

47. Prove: If the sequence $\langle a_n \rangle$ converges to $b \in \mathbf{R}$, then the sequence $\langle |a_n - b| \rangle$ converges to 0.

48. Prove: If the sequence $\langle a_n \rangle$ converges to 0, and the sequence $\langle b_n \rangle$ is bounded, then the seq $\langle a_n b_n \rangle$ also converges to 0.

49. Prove: If $a_n \to a$ and $b_n \to b$, then the sequence $\langle a_n + b_n \rangle$ converges to $a + b$.

50. Prove: If $a_n \to a$ and $b_n \to b$, then the sequence $\langle a_n b_n \rangle$ converges to ab.