1. Let $\sim$ be the equivalence relation on $X = \mathbb{N} \times \mathbb{N}$ given by $(a,b) \sim (c,d)$ means $a+d = b+c$.

Define $+$ on the set of equivalence classes $X/\sim$ by $[(a,b)] + [(c,d)] = [(a+c, b+d)]$.

Show this is well defined, commutative and associative.

2. Use Schroeder-Bernstein to prove

$$(0,1) \sim \prod_{i=1}^{\infty} \{0,1\} \quad \text{so} \quad c = 2^{\aleph_0}$$

Hint: use the decimal expansion for each element in $(0,1)$.

3. Prove that $X \sim Y \Rightarrow \mathcal{P}(X) \sim \mathcal{P}(Y)$.