MA 1126 Assignment 3

1 For the following show $\sim$ is an equivalence relation and identify $X/\sim$

(a) $X = \mathbb{R} \times \mathbb{R}$ $(x_1, y_1) \sim (x_2, y_2)$ means $x_1 = x_2$

(b) $X = \mathbb{N} \times \mathbb{N}$, $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
$(a, b) \sim (c, d)$ means $a + d = b + c$

(c) $f : [0, 1] \to \mathbb{C}$ by $f(x) = e^{2\pi i x}$ $x_1 \sim x_2$ means $f(x_1) = f(x_2)$

2. Prove $f(A \cap B) = f(A) \cap f(B)$ $\forall A, B \subset X \iff f$ is 1-1

~~3. Prove $f(A) \cup f(B) = f(A)$~~

3. Prove $f(A \setminus B) = f(A) \setminus f(B)$ $\forall A, B \subset X \iff f$ is 1-1.

4. Define $\chi_A(x) = 1$ if $x \in A$
$= 0$ if $x \notin A$
called the characteristic function of A.

Show (i) $\chi_{A \cap B} = \chi_A \cdot \chi_B$

(ii) $\chi_{A \cup B} = \chi_A + \chi_B - \chi_{A \cap B}$.