1. (a) Use a true/false table to prove $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$. Is there a name for this statement?

   (b) Complete and prove the following $A \vee (B \wedge C) \equiv$

   (c) Is the following a valid argument? $P \vee Q \vee R$, and $Q \wedge R$, therefore P. Be sure to prove your answer. 7

   (d) State Zorn's Lemma and the Axiom of Choice and the Well Ordering Principle.

2. (a) Define what it means for two sets to have the same cardinal number. Define countable set. Prove that $\mathbb{Q}$ is countable but $\mathbb{R}$ is not.

   (b) State the Schroeder-Bernstein Theorem and Cantor's Theorem. Prove Cantor's Theorem.

   (c) Prove $\prod_{1,\infty}\{0,1\} \sim \mathbb{P}(\mathbb{N})$. 8

3. (a) Define open set in $\mathbb{R}$, and closed set. Prove that a set is closed if and only if it contains all its accumulation points.

   (b) Define Cauchy sequence. State and prove the Nested Intervals Theorem. State the least upper bound axiom for $\mathbb{R}$. State The Bolzano- Weierstrass Theorem for $\mathbb{R}$. What are the connections? 4

4. (a) Show that $f : \mathbb{R} \to \mathbb{R}$ is continuous iff for every open set $O, f^{-1}(O)$ is open.

   (b) Prove that the continuous image of a connected subset of $\mathbb{R}$ is connected.

   (c) Use part (b) to prove the Intermediate Value Theorem. 6