1. (a) Define function, and inverse function.

   (b) Use the definition of a limit to calculate $\lim_{x\to 2} 2x^3 + 3x^2 - 2$.

   (c) Define $\lim_{x\to a^+} f(x) = L$.

   (d) Prove that limits are unique.

2. (a) Derive the formula for the linear approximation to $f(x)$, and use it to find $\sqrt{47}$ to two decimal places.

   (b) Find $\frac{dy}{dx}$ if

   i. $y = \dfrac{\sin x \ln x \cos x}{x^2}$.

   ii. $x^2y^3 + \sin(x^2y) = \exp(xy^3)$

   iii. $y = \tan^{-1} x$.

   (c) Find where the function $x \exp x$ is increasing, decreasing, concave up, concave down, has local extrema, and points of inflection. Draw a rough sketch of the function.

3. (a) State The Generalised Mean Value Theorem. State the Extreme Value Theorem.

   (b) Solve the following

   i. $x\dfrac{dy}{dx} = y^2 + 1,\ y(1) = 0.$

   ii. $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = 0$

   iii. $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = x + \sin x$

   (c) Find the area of the largest rectangle that can be inscribed in a circle of radius r

4. (a) How is the Riemann Integral $\int_a^b f(x)dx$ defined?

   (b) Integrate the following.

   i. $\int \frac{1}{\sqrt{4-x^2}}dx$

   ii. $\int x \ln x dx$

   iii. $\int \sin^2 x \cos^3 x dx$

   iv. $\int \frac{3x^3 - x^2 + 6x - 4}{(x^2+1)(x^2+2)}$

5. (a) Find the length of the curve $y = \int_1^x \sqrt{\sqrt{t} - 1}dt$ from x=1 to x=16.

   (b) Find the area enclosed between $y = x^2$ and the line $y = x + 4$..

   (c) The bounded region between $y = x$, $x = 2$ and above the x-axis is rotated about the line $x = 3$. Find the volume first using washers and then using cylindrical shells.

6. (a) Define $\sum_{n=1}^{\infty} a_n = L$.

   (b) Do the following series converge or diverge? Give reasons.

   i. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

   ii. $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$

   (c) Find the Taylor Series for $f(x) = \exp(x)$ about $x = 0$. How would you determine where this series converges to $\exp x$?

   (d) What is the integral test?