UNIVERSITY OF DUBLIN

XMA11231

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS AND SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Michaelmas Term 2012

MA1123 — ANALYSIS I

Tuesday, August 28 LUCE UPPER 9:30 — 13:00

Prof.D.O'Donovan

ANSWER ALL QUESTIONS:

All questions carry equal marks.

Formulae & Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. (a) Define function

   (b) Define $\lim_{x \to a} f(x) = L$

   (c) Let $f(x)$ be defined as follows

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 \leq x \leq 2 \\ 2 & \text{if } x = 2 \end{cases}$$

   At what points c in the domain does the $\lim_{x \to c} f(x)$ exist? What about the right and left hand limits?

   (d) Use the definition of limit to show $\lim_{x \to -1} x^2 + 2x + 1 = 0$

2. (a) Let $f(x) = x$ if $x \leq 0$, and $f(x) = x^2$ if $x \geq 0$, find $f'(0)$ or show that it does not exist.

   (b) If $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$, show $\lim_{x \to a}(f - g)(x) = L_1 - L_2$

   (c) Find the following limits or explain why they do not exist

$$\lim_{x \to 0} \frac{\sin 3x}{5x}, \quad \lim_{x \to \infty} \frac{\ln x}{x^2}, \quad \lim_{x \to 0} \sin \frac{1}{x}, \quad \lim_{x \to 0} \frac{x^2 + 7x + 1}{2x^2 + 5}$$

   (d) Find the quadratic approximation to $\cos(0.1)$

3. (a) Find $\dfrac{dy}{dx}$ if

      i. $y = \sqrt[3]{\cos^2(x^2 + 2x + 1)}$

      ii. $y = x^3 \exp(x) \cos(x) \sin(x)$

      iii. $x^3 y + y^2 x + \cos(xy) = 1$

      iv. $x = \cos t^2, y = \sin(t^2 + t)$

   (b) Let $f(x) = \dfrac{8}{x^2 - 4}$. Find where $f(x)$ is increasing, decreasing, concave up, concave down, has local extrema, and points of inflection. Use this information to sketch the function.

4. (a) Prove Rolle's Theorem and state the Mean Value Theorem.

   (b) Prove that if $f'(x) = g'(x)$, for all x, then $f(x) = g(x) +$ constant.

   (c) How fast does the water level drop when a cylindical tank is drained at the rate of 3 litres/second?

5. (a) State how $\int_a^b f(x)\,dx$ is defined in terms of Riemann Sums

   (b) Integrate the following.

   i. $\displaystyle\int \exp x \cos x \, dx$

   ii. $\displaystyle\int \sin x^2 \cos x^2 \, dx$

   iii. $\displaystyle\int \frac{1}{x^2+3x+1}\, dx$

   iv. $\displaystyle\int \frac{x^2+1}{(x-1)}\, dx$

   v. $\displaystyle\int \frac{1}{(x-1)^3(x-2)^2}\, dx$

6. (a) Find the area of the region bounded by $y = x^2$, $y = x + 2$ and $y = -x + 4$

   (b) Find the volume of the solid of revolution gotten by revolving the region bounded by $y = x^2, y = 0$, and $x = 1$ about the y-axis, first by the method of disks, and then by the method of cylindrical shells.