Assignment 7 MA1123 Due Monday 30th.

1. p353 12,21-24,28
2. p362-363 16,20,21-24,34,43
3. p370-371 15-18,26,28
4. p375-376 9-12,26
5. p399 22

10th Edition of Anton.

✓ QUICK CHECK EXERCISES 5.1

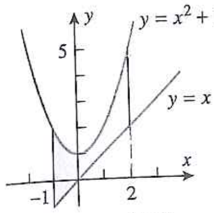
(See page 355 for answers.)

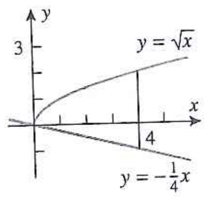
1. An integral expression for the area of the region between the curves $y = 20 - 3x^2$ and $y = 3\sqrt{x}$ and bounded on the sides by $x = 0$ and $x = 2$ is ______.
2. An integral expression for the area of the parallelogram bounded by $y = 2x + 8$, $y = 2x - 3$, $x = -1$, and $x = 5$ is ______. The value of this integral is ______.
3. (a) The points of intersection for the circle $x^2 + y^2 = 4$ and the line $y = x + 2$ are ______ and ______.
   (b) Expressed as a definite integral with respect to x, ______ gives the area of the region inside the circle $x^2 + y^2 = 4$ and above the line $y = x + 2$.
   (c) Expressed as a definite integral with respect to y, ______ gives the area of the region described in part (b).
4. The area of the region enclosed by the curves $y = x^2$ and $y = \sqrt[3]{x}$ is ______.

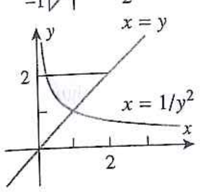
EXERCISE SET 5.1

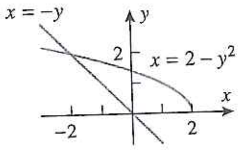
Graphing Utility [C] CAS

1–4 Find the area of the shaded region.

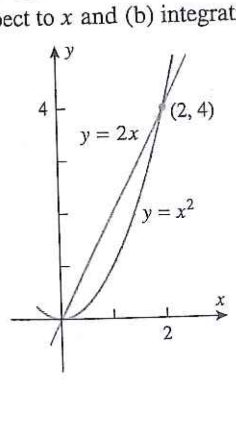
1.


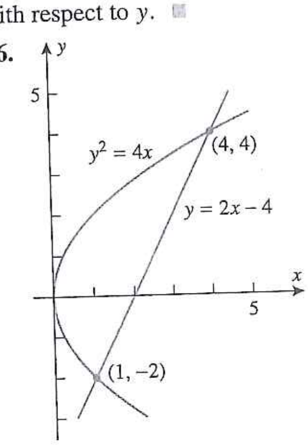
2.


3.


4.


5–6 Find the area of the shaded region by (a) integrating with respect to x and (b) integrating with respect to y.

5.


6.


7–14 Sketch the region enclosed by the curves and find its area.

7. $y = x^2,\ y = \sqrt{x},\ x = \frac{1}{4},\ x = 1$
8. $y = x^3 - 4x,\ y = 0,\ x = 0,\ x = 2$
9. $y = \cos 2x,\ y = 0,\ x = \pi/4,\ x = \pi/2$
10. $y = \sec^2 x,\ y = 2,\ x = -\pi/4,\ x = \pi/4$
11. $x = \sin y,\ x = 0,\ y = \pi/4,\ y = 3\pi/4$
12. $x^2 = y,\ x = y - 2$
13. $y = 2 + |x - 1|,\ y = -\frac{1}{5}x + 7$
14. $y = x,\ y = 4x,\ y = -x + 2$

15–20 Use a graphing utility, where helpful, to find the area of the region enclosed by the curves.

15. $y = x^3 - 4x^2 + 3x,\ y = 0$
16. $y = x^3 - 2x^2,\ y = 2x^2 - 3x$
17. $y = \sin x,\ y = \cos x,\ x = 0,\ x = 2\pi$
18. $y = x^3 - 4x,\ y = 0$
19. $x = y^3 - y,\ x = 0$
20. $x = y^3 - 4y^2 + 3y,\ x = y^2 - y$

21–24 True–False Determine whether the statement is true or false. Explain your answer. [In each exercise, assume that f and g are distinct continuous functions on $[a, b]$ and that A denotes the area of the region bounded by the graphs of $y = f(x)$, $y = g(x)$, $x = a$, and $x = b$.]

21. If f and g differ by a positive constant c, then $A = c(b - a)$.
22. If
$$\int_a^b [f(x) - g(x)]\,dx = -3$$
then $A = 3$.
23. If
$$\int_a^b [f(x) - g(x)]\,dx = 0$$
then the graphs of $y = f(x)$ and $y = g(x)$ cross at least once on $[a, b]$.
24. If
$$A = \left|\int_a^b [f(x) - g(x)]\,dx\right|$$
then the graphs of $y = f(x)$ and $y = g(x)$ don't cross on $[a, b]$.

25. [C] Use a CAS to find the area enclosed by $y = 3 - 2x$ and $y = x^6 + 2x^5 - 3x^4 + x^2$.
26. [C] Use a CAS to find the exact area enclosed by the curves $y = x^5 - 2x^3 - 3x$ and $y = x^3$.
27. Find a horizontal line $y = k$ that divides the area between $y = x^2$ and $y = 9$ into two equal parts.
28. Find a vertical line $x = k$ that divides the area enclosed by $x = \sqrt{y}$, $x = 2$, and $y = 0$ into two equal parts.

✔ QUICK CHECK EXERCISES 5.2 (See page 365 for answers.)

1. A solid S extends along the x-axis from $x = 1$ to $x = 3$. For x between 1 and 3, the cross-sectional area of S perpendicular to the x-axis is $3x^2$. An integral expression for the volume of S is ______. The value of this integral is ______.

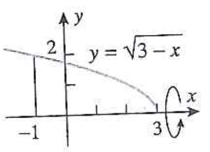
2. A solid S is generated by revolving the region between the x-axis and the curve $y = \sqrt{\sin x}$ $(0 \le x \le \pi)$ about the x-axis.
   (a) For x between 0 and π, the cross-sectional area of S perpendicular to the x-axis at x is $A(x) =$ ______.
   (b) An integral expression for the volume of S is ______.
   (c) The value of the integral in part (b) is ______.

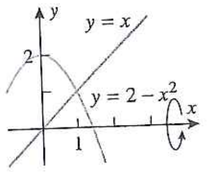
3. A solid S is generated by revolving the region enclosed by the line $y = 2x + 1$ and the curve $y = x^2 + 1$ about the x-axis.
   (a) For x between ______ and ______, the cross-sectional area of S perpendicular to the x-axis at x is $A(x) =$ ______.
   (b) An integral expression for the volume of S is ______.

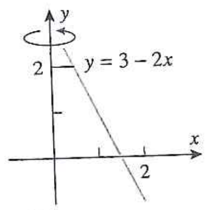
4. A solid S is generated by revolving the region enclosed by the line $y = x + 1$ and the curve $y = x^2 + 1$ about the y-axis.
   (a) For y between ______ and ______, the cross-sectional area of S perpendicular to the y-axis at y is $A(y) =$ ______.
   (b) An integral expression for the volume of S is ______.

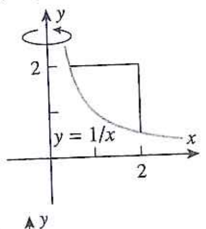
EXERCISE SET 5.2 [C] CAS

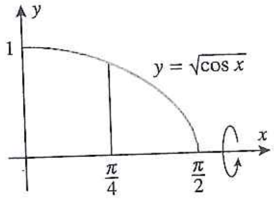
1–8 Find the volume of the solid that results when the shaded region is revolved about the indicated axis.

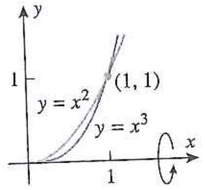
1.


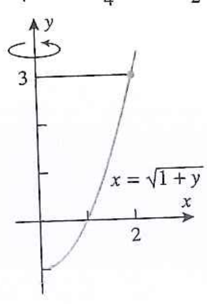
2.


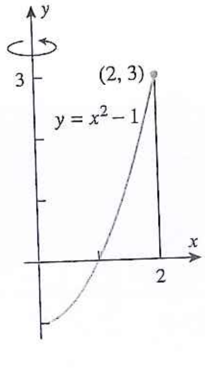
3.


4.


5.


6.


7.


8.


9. Find the volume of the solid whose base is the region bounded between the curve $y = x^2$ and the x-axis from $x = 0$ to $x = 2$ and whose cross sections taken perpendicular to the x-axis are squares.

10. Find the volume of the solid whose base is the region bounded between the curve $y = \sec x$ and the x-axis from $x = \pi/4$ to $x = \pi/3$ and whose cross sections taken perpendicular to the x-axis are squares.

11–14 Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.

11. $y = \sqrt{25 - x^2}$, $y = 3$

12. $y = 9 - x^2$, $y = 0$

13. $x = \sqrt{y}$, $x = y/4$

14. $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$
[*Hint:* Use the identity $\cos 2x = \cos^2 x - \sin^2 x$.]

15. Find the volume of the solid whose base is the region bounded between the curve $y = x^3$ and the y-axis from $y = 0$ to $y = 1$ and whose cross sections taken perpendicular to the y-axis are squares.

16. Find the volume of the solid whose base is the region enclosed between the curve $x = 1 - y^2$ and the y-axis and whose cross sections taken perpendicular to the y-axis are squares.

17–20 Find the volume of the solid that results when the region enclosed by the given curves is revolved about the y-axis.

17. $x = \csc y$, $y = \pi/4$, $y = 3\pi/4$, $x = 0$

18. $y = x^2$, $x = y^2$

19. $x = y^2$, $x = y + 2$

20. $x = 1 - y^2$, $x = 2 + y^2$, $y = -1$, $y = 1$

21–24 True–False Determine whether the statement is true or false. Explain your answer. [In these exercises, assume that a solid S of volume V is bounded by two parallel planes perpendicular to the x-axis at $x = a$ and $x = b$ and that for each x in $[a, b]$, $A(x)$ denotes the cross-sectional area of S perpendicular to the x-axis.]

21. If each cross section of S perpendicular to the x-axis is a square, then S is a rectangular parallelepiped (i.e., is box shaped).

22. If each cross section of S is a disk or a washer, then S is a solid of revolution.

23. If x is in centimeters (cm), then $A(x)$ must be a quadratic function of x, since units of $A(x)$ will be square centimeters (cm^2).

24. The average value of $A(x)$ on the interval $[a, b]$ is given by $V/(b - a)$.

25. Find the volume of the solid that results when the region above the x-axis and below the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > 0, b > 0)$$

is revolved about the x-axis.

26. Let V be the volume of the solid that results when the region enclosed by $y = 1/x$, $y = 0$, $x = 2$, and $x = b$ $(0 < b < 2)$ is revolved about the x-axis. Find the value of b for which $V = 3$.

27. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x+1}$, $y = \sqrt{2x}$, and $y = 0$ is revolved about the x-axis. [*Hint:* Split the solid into two parts.]

28. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and $y = 0$ is revolved about the x-axis. [*Hint:* Split the solid into two parts.]

FOCUS ON CONCEPTS

29. Suppose that f is a continuous function on $[a, b]$, and let R be the region between the curve $y = f(x)$ and the line $y = k$ from $x = a$ to $x = b$. Using the method of disks, derive with explanation a formula for the volume of a solid generated by revolving R about the line $y = k$. State and explain additional assumptions, if any, that you need about f for your formula.

30. Suppose that v and w are continuous functions on $[c, d]$, and let R be the region between the curves $x = v(y)$ and $x = w(y)$ from $y = c$ to $y = d$. Using the method of washers, derive with explanation a formula for the volume of a solid generated by revolving R about the line $x = k$. State and explain additional assumptions, if any, that you need about v and w for your formula.

31. Consider the solid generated by revolving the shaded region in Exercise 1 about the line $y = 2$.
(a) Make a conjecture as to which is larger: the volume of this solid or the volume of the solid in Exercise 1. Explain the basis of your conjecture.
(b) Check your conjecture by calculating this volume and comparing it to the volume obtained in Exercise 1.

32. Consider the solid generated by revolving the shaded region in Exercise 4 about the line $x = 2.5$.
(a) Make a conjecture as to which is larger: the volume of this solid or the volume of the solid in Exercise 4. Explain the basis of your conjecture.
(b) Check your conjecture by expressing the difference in the two volumes as a single definite integral. [*Hint:* Sketch the graph of the integrand.]

33. Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 9$ is revolved about the line $x = 9$.

34. Find the volume of the solid that results when the region in Exercise 33 is revolved about the line $y = 3$.

35. Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

36. Find the volume of the solid that results when the region in Exercise 35 is revolved about the line $x = -1$.

37. Find the volume of the solid that results when the region enclosed by $y = x^2$ and $y = x^3$ is revolved about the line $x = 1$.

38. Find the volume of the solid that results when the region in Exercise 37 is revolved about the line $y = -1$.

39. A nose cone for a space reentry vehicle is designed so that a cross section, taken x ft from the tip and perpendicular to the axis of symmetry, is a circle of radius $\frac{1}{4}x^2$ ft. Find the volume of the nose cone given that its length is 20 ft.

40. A certain solid is 1 ft high, and a horizontal cross section taken x ft above the bottom of the solid is an annulus of inner radius x^2 ft and outer radius $\sqrt{x}$ ft. Find the volume of the solid.

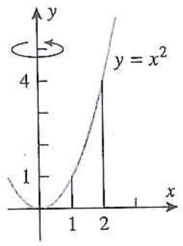
41. Find the volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the x-axis are squares.

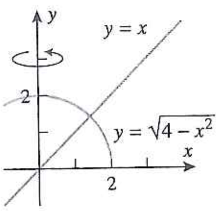
42. The base of a certain solid is the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.

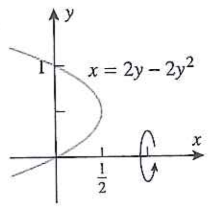
43. In parts (a)–(c) find the volume of the solid whose base is enclosed by the circle $x^2 + y^2 = 1$ and whose cross sections taken perpendicular to the x-axis are as indicated. *(cont.)*

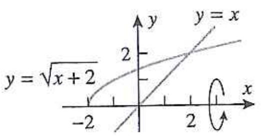
EXERCISE SET 5.3 [C] CAS

1–4 Use cylindrical shells to find the volume of the solid generated when the shaded region is revolved about the indicated axis.

1.


2.


3.


4.


5–10 Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about the y-axis.

5. $y = x^3,\ x = 1,\ y = 0$

6. $y = \sqrt{x},\ x = 4,\ x = 9,\ y = 0$

7. $y = 1/x,\ y = 0,\ x = 1,\ x = 3$

8. $y = \cos(x^2),\ x = 0,\ x = \frac{1}{2}\sqrt{\pi},\ y = 0$

9. $y = 2x - 1,\ y = -2x + 3,\ x = 2$

10. $y = 2x - x^2,\ y = 0$

11–14 Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about the x-axis.

11. $y^2 = x,\ y = 1,\ x = 0$

12. $x = 2y,\ y = 2,\ y = 3,\ x = 0$

13. $y = x^2,\ x = 1,\ y = 0$ **14.** $xy = 4,\ x + y = 5$

15–18 True–False Determine whether the statement is true or false. Explain your answer.

15. The volume of a cylindrical shell is equal to the product of the thickness of the shell with the surface area of a cylinder whose height is that of the shell and whose radius is equal to the average of the inner and outer radii of the shell.

16. The method of cylindrical shells is a special case of the method of integration of cross-sectional area that was discussed in Section 5.2.

17. In the method of cylindrical shells, integration is over an interval on a coordinate axis that is *perpendicular* to the axis of revolution of the solid.

18. The Riemann sum approximation

$$V \approx \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k \quad \left(\text{where } x_k^* = \frac{x_k + x_{k-1}}{2}\right)$$

for the volume of a solid of revolution is exact when f is a constant function.

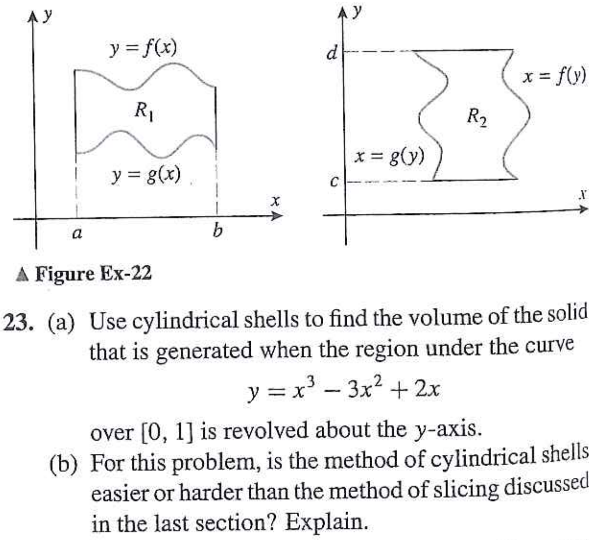
[C] **19.** Use a CAS to find the volume of the solid generated when the region enclosed by $y = \sin x$ and $y = 0$ for $0 \le x \le \pi$ is revolved about the y-axis.

[C] **20.** Use a CAS to find the volume of the solid generated when the region enclosed by $y = \cos x$, $y = 0$, and $x = 0$ for $0 \le x \le \pi/2$ is revolved about the y-axis.

[C] **21.** Consider the region to the right of the y-axis, to the left of the vertical line $x = k$ $(0 < k < \pi)$, and between the curve $y = \sin x$ and the x-axis. Use a CAS to estimate the value of k so that the solid generated by revolving the region about the y-axis has a volume of 8 cubic units.

FOCUS ON CONCEPTS

22. Let R_1 and R_2 be regions of the form shown in the accompanying figure. Use cylindrical shells to find a formula for the volume of the solid that results when
(a) region R_1 is revolved about the y-axis
(b) region R_2 is revolved about the x-axis.



▲ **Figure Ex-22**

23. (a) Use cylindrical shells to find the volume of the solid that is generated when the region under the curve
$$y = x^3 - 3x^2 + 2x$$
over $[0, 1]$ is revolved about the y-axis.
(b) For this problem, is the method of cylindrical shells easier or harder than the method of slicing discussed in the last section? Explain.

24. Let f be continuous and nonnegative on $[a, b]$, and let R be the region that is enclosed by $y = f(x)$ and $y = 0$ for $a \le x \le b$. Using the method of cylindrical shells, derive with explanation a formula for the volume of the solid generated by revolving R about the line $x = k$, where $k \le a$.

25–26 Using the method of cylindrical shells, set up but do not evaluate an integral for the volume of the solid generated when the region R is revolved about (a) the line $x = 1$ and (b) the line $y = -1$.

25. R is the region bounded by the graphs of $y = x$, $y = 0$, and $x = 1$.

26. R is the region in the first quadrant bounded by the graphs of $y = \sqrt{1 - x^2}$, $y = 0$, and $x = 0$.

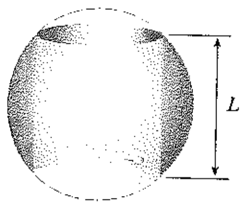
27. Use cylindrical shells to find the volume of the solid that is generated when the region that is enclosed by $y = 1/x^3$, $x = 1$, $x = 2$, $y = 0$ is revolved about the line $x = -1$.

28. Use cylindrical shells to find the volume of the solid that is generated when the region that is enclosed by $y = x^3$, $y = 1$, $x = 0$ is revolved about the line $y = 1$.

29. Use cylindrical shells to find the volume of the cone generated when the triangle with vertices $(0, 0)$, $(0, r)$, $(h, 0)$, where $r > 0$ and $h > 0$, is revolved about the x-axis.

30. The region enclosed between the curve $y^2 = kx$ and the line $x = \frac{1}{4}k$ is revolved about the line $x = \frac{1}{2}k$. Use cylindrical shells to find the volume of the resulting solid. (Assume $k > 0$.)

31. As shown in the accompanying figure, a cylindrical hole is drilled all the way through the center of a sphere. Show that the volume of the remaining solid depends only on the length L of the hole, not on the size of the sphere.



◀ Figure Ex-31

32. Use cylindrical shells to find the volume of the torus obtained by revolving the circle $x^2 + y^2 = a^2$ about the line $x = b$, where $b > a > 0$. [*Hint:* It may help in the integration to think of an integral as an area.]

33. Let V_x and V_y be the volumes of the solids that result when the region enclosed by $y = 1/x$, $y = 0$, $x = \frac{1}{2}$, and $x = b$ $\left(b > \frac{1}{2}\right)$ is revolved about the x-axis and y-axis, respectively. Is there a value of b for which $V_x = V_y$?

34. **Writing** Faced with the problem of computing the volume of a solid of revolution, how would you go about deciding whether to use the method of disks/washers or the method of cylindrical shells?

35. **Writing** With both the method of disks/washers and with the method of cylindrical shells, we integrate an "area" to get the volume of a solid of revolution. However, these two approaches differ in very significant ways. Write a brief paragraph that discusses these differences.

✔ QUICK CHECK ANSWERS 5.3

1. (a) $2\pi x(1 + \sqrt{x})$ (b) $\int_1^4 2\pi x(1 + \sqrt{x})\,dx$ 2. (a) $2\pi(5 - x)(1 + \sqrt{x})$ (b) $\int_1^4 2\pi(5 - x)(1 + \sqrt{x})\,dx$
3. $\int_0^4 2\pi y[4 - (y - 2)^2]\,dy$

5.4 LENGTH OF A PLANE CURVE

In this section we will use the tools of calculus to study the problem of finding the length of a plane curve.

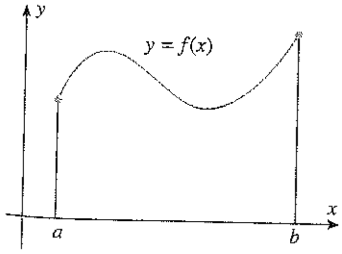


Figure 5.4.1

ARC LENGTH

Our first objective is to define what we mean by the ***length*** (also called the ***arc length***) of a plane curve $y = f(x)$ over an interval $[a, b]$ (Figure 5.4.1). Once that is done we will be able to focus on the problem of computing arc lengths. To avoid some complications that would otherwise occur, we will impose the requirement that f' be continuous on $[a, b]$, in which case we will say that $y = f(x)$ is a ***smooth curve*** on $[a, b]$ or that f is a ***smooth function*** on $[a, b]$. Thus, we will be concerned with the following problem.

nd the exact arc length of the curve over the interval.

$= 3x^{3/2} - 1$ from $x = 0$ to $x = 1$

$= \frac{1}{3}(y^2 + 2)^{3/2}$ from $y = 0$ to $y = 1$

$= x^{2/3}$ from $x = 1$ to $x = 8$

$= (x^6 + 8)/(16x^2)$ from $x = 2$ to $x = 3$

$y = y^4 + 48$ from $y = 2$ to $y = 4$

$= \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$ from $y = 1$ to $y = 4$

rue–False Determine whether the statement is true or
xplain your answer.

e graph of $y = \sqrt{1 - x^2}$ is a smooth curve on $[-1, 1]$.

e approximation

$$L \approx \sum_{k=1}^{n} \sqrt{(\Delta x_k)^2 + [f(x_k) - f(x_{k-1})]^2}$$

arc length is not expressed in the form of a Riemann
m.

e approximation

$$L \approx \sum_{k=1}^{n} \sqrt{1 + [f'(x_k^*)]^2}\,\Delta x_k$$

r arc length is exact when f is a linear function of x.

our definition of the arc length for the graph of $y = f(x)$,
e need $f'(x)$ to be a continuous function in order for f to
tisfy the hypotheses of the Mean-Value Theorem (3.8.2).

S ON CONCEPTS

Consider the curve $y = x^{2/3}$.
(a) Sketch the portion of the curve between $x = -1$ and $x = 8$.
(b) Explain why Formula (4) cannot be used to find the arc length of the curve sketched in part (a).
(c) Find the arc length of the curve sketched in part (a).

The curve segment $y = x^2$ from $x = 1$ to $x = 2$ may also be expressed as the graph of $x = \sqrt{y}$ from $y = 1$ to $y = 4$. Set up two integrals that give the arc length of this curve segment, one by integrating with respect to x, and the other by integrating with respect to y. Demonstrate a substitution that verifies that these two integrals are equal.

Consider the curve segments $y = x^2$ from $x = \frac{1}{2}$ to $x = 2$ and $y = \sqrt{x}$ from $x = \frac{1}{4}$ to $x = 4$.
(a) Graph the two curve segments and use your graphs to explain why the lengths of these two curve segments should be equal.
(b) Set up integrals that give the arc lengths of the curve segments by integrating with respect to x. Demonstrate a substitution that verifies that these two integrals are equal.
(c) Set up integrals that give the arc lengths of the curve segments by integrating with respect to y.

(d) Approximate the arc length of each curve segment using Formula (2) with $n = 10$ equal subintervals.
(e) Which of the two approximations in part (d) is more accurate? Explain.
(f) Use the midpoint approximation with $n = 10$ subintervals to approximate each arc length integral in part (b).
(g) Use a calculating utility with numerical integration capabilities to approximate the arc length integrals in part (b) to four decimal places.

16. Follow the directions of Exercise 15 for the curve segments $y = x^{8/3}$ from $x = 10^{-3}$ to $x = 1$ and $y = x^{3/8}$ from $x = 10^{-8}$ to $x = 1$.

17. Follow the directions of Exercise 15 for the curve segment $y = 1 + 1/x$ from $x = 1$ to $x = 3$ and for the curve segment $y = 1/(x - 1)$ from $x = 4/3$ to $x = 2$.

18. Let $y = f(x)$ be a smooth curve on the closed interval $[a, b]$. Prove that if m and M are nonnegative numbers such that $m \le |f'(x)| \le M$ for all x in $[a, b]$, then the arc length L of $y = f(x)$ over the interval $[a, b]$ satisfies the inequalities

$$(b - a)\sqrt{1 + m^2} \le L \le (b - a)\sqrt{1 + M^2}$$

19. Use the result of Exercise 18 to show that the arc length L of $y = \sec x$ over the interval $0 \le x \le \pi/3$ satisfies

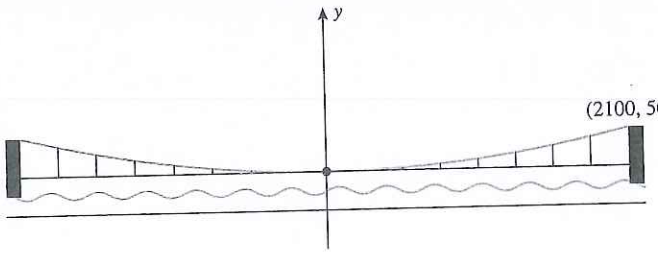
$$\frac{\pi}{3} \le L \le \frac{\pi}{3}\sqrt{13}$$

C **20.** A basketball player makes a successful shot from the free throw line. Suppose that the path of the ball from the moment of release to the moment it enters the hoop is described by

$$y = 2.15 + 2.09x - 0.41x^2, \quad 0 \le x \le 4.6$$

where x is the horizontal distance (in meters) from the point of release, and y is the vertical distance (in meters) above the floor. Use a CAS or a scientific calculator with a numerical integration capability to approximate the distance the ball travels from the moment it is released to the moment it enters the hoop. Round your answer to two decimal places.

C **21.** The central span of the Golden Gate Bridge in California is 4200 ft long and is suspended from cables that rise 500 ft above the roadway on either side. Approximately how long is the portion of a cable that lies between the support towers on one side of the roadway? [*Hint:* As suggested by the accompanying figure on the next page, assume the cable is modeled by a parabola $y = ax^2$ that passes through the point (2100, 500). Use a CAS or a calculating utility with a numerical integration capability to approximate the length of the cable. Round your answer to the nearest foot.]



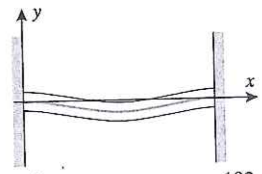
▲ Figure Ex-21

C 22. As shown in the accompanying figure, a horizontal beam with dimensions 2 in × 6 in × 16 ft is fixed at both ends and is subjected to a uniformly distributed load of 120 lb/ft. As a result of the load, the centerline of the beam undergoes a deflection that is described by

$$y = -1.67 \times 10^{-8}(x^4 - 2Lx^3 + L^2x^2)$$

($0 \le x \le 192$), where $L = 192$ in is the length of the unloaded beam, x is the horizontal distance along the beam measured in inches from the left end, and y is the deflection of the centerline in inches.

(a) Graph y versus x for $0 \le x \le 192$.
(b) Find the maximum deflection of the centerline.
(c) Use a CAS or a calculator with a numerical integration capability to find the length of the centerline of the loaded beam. Round your answer to two decimal places.



◀ Figure Ex-22

C 23. A golfer makes a successful chip shot to the green. Suppose that the path of the ball from the moment it is struck to the moment it hits the green is described by

$$y = 12.54x - 0.41x^2$$

where x is the horizontal distance (in yards) from the point where the ball is struck, and y is the vertical distance (in yards) above the fairway. Use a CAS or a calculating utility with a numerical integration capability to find the distance the ball travels from the moment it is struck to the moment it hits the green. Assume that the fairway and green are at the same level and round your answer to two decimal places.

24–30 These exercises assume familiarity with the basic concepts of parametric curves. If needed, an introduction to this material is provided in Web Appendix I.

C 24. Assume that no segment of the curve

$$x = x(t), \quad y = y(t), \quad (a \le t \le b)$$

is traced more than once as t increases from a to b. Divide the interval $[a, b]$ into n subintervals by inserting points $t_1, t_2, \ldots, t_{n-1}$ between $a = t_0$ and $b = t_n$. Let L denote the arc length of the curve. Give an informal argument for the approximation

$$L \approx \sum_{k=1}^{n} \sqrt{[x(t_k) - x(t_{k-1})]^2 + [y(t_k) - y(t_{k-1})]^2}$$

If dx/dt and dy/dt are continuous functions for $a \le t \le b$, then it can be shown that as $\max \Delta t_k \to 0$, this sum converges to

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

25–28 Use the arc length formula from Exercise 24 to find the arc length of the curve.

25. $x = \frac{1}{3}t^3,\ y = \frac{1}{2}t^2 \quad (0 \le t \le 1)$
26. $x = (1+t)^2,\ y = (1+t)^3 \quad (0 \le t \le 1)$
27. $x = \cos 2t,\ y = \sin 2t \quad (0 \le t \le \pi/2)$
28. $x = \cos t + t\sin t,\ y = \sin t - t\cos t \quad (0 \le t \le \pi)$

C 29. (a) Show that the total arc length of the ellipse

$$x = 2\cos t, \quad y = \sin t \qquad (0 \le t \le 2\pi)$$

is given by

$$4\int_0^{\pi/2} \sqrt{1 + 3\sin^2 t}\, dt$$

(b) Use a CAS or a scientific calculator with a numerical integration capability to approximate the arc length in part (a). Round your answer to two decimal places.
(c) Suppose that the parametric equations in part (a) describe the path of a particle moving in the xy-plane, where t is time in seconds and x and y are in centimeters. Use a CAS or a scientific calculator with a numerical integration capability to approximate the distance traveled by the particle from $t = 1.5$ s to $t = 4.8$ s. Round your answer to two decimal places.

30. Show that the total arc length of the ellipse $x = a\cos t$, $y = b\sin t$, $0 \le t \le 2\pi$ for $a > b > 0$ is given by

$$4a\int_0^{\pi/2} \sqrt{1 - k^2\cos^2 t}\, dt$$

where $k = \sqrt{a^2 - b^2}/a$.

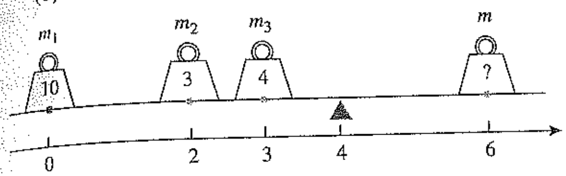
31. **Writing** In our discussion of Arc Length Problem 5.4.1, we derived the approximation

$$L \approx \sum_{k=1}^{n} \sqrt{1 + [f'(x_k^*)]^2}\, \Delta x_k$$

Discuss the geometric meaning of this approximation. (Be sure to address the appearance of the derivative f'.)

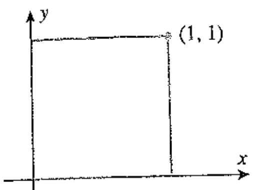
32. **Writing** Give examples in which Formula (4) for arc length cannot be applied directly, and describe how you would go about finding the arc length of the curve in each case. (Discuss both the use of alternative formulas and the use of numerical methods.)

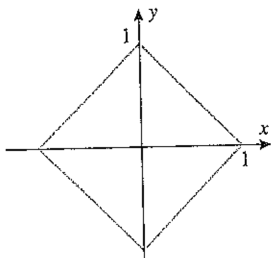
2. Masses $m_1 = 10, m_2 = 3, m_3 = 4$, and m are positioned on a weightless beam, with the fulcrum positioned at point 4, as shown in the accompanying figure.
   (a) Suppose that $m = 14$. Without computing the sum of the moments about 4, determine whether the sum is positive, zero, or negative. Explain.
   (b) For what value of m is the beam in equilibrium?

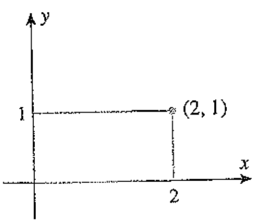


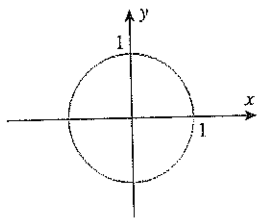
▲ Figure Ex-2

3–6 Find the centroid of the region by inspection and confirm your answer by integrating.

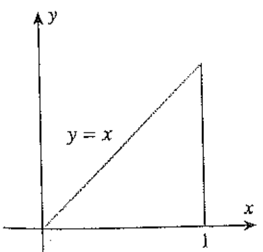
3.


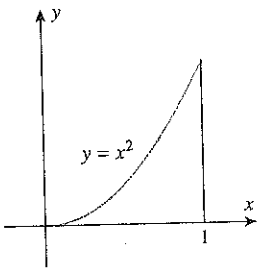
4.


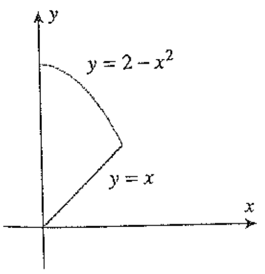
5.


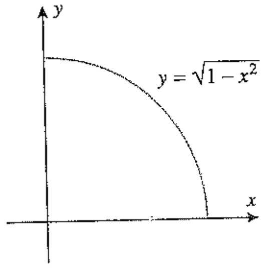
6.


7–18 Find the centroid of the region.

7.


8.


9.


10.


11. The triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 1)$.

12. The triangle with vertices $(0, 0)$, $(1, 1)$, and $(2, 0)$.

13. The region bounded by the graphs of $y = x^2$ and $x + y = 6$.

14. The region bounded on the left by the y-axis, on the right by the line $x = 2$, below by the parabola $y = x^2$, and above by the line $y = x + 6$.

15. The region bounded by the graphs of $y = x^2$ and $y = x + 2$.

16. The region bounded by the graphs of $y = x^2$ and $y = 1$.

17. The region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$.

18. The region bounded by the graphs of $y = \sqrt{x}$ and $y = x^3$.

FOCUS ON CONCEPTS

19. Use symmetry considerations to argue that the centroid of an isosceles triangle lies on the median to the base of the triangle.

20. Use symmetry considerations to argue that the centroid of an ellipse lies at the intersection of the major and minor axes of the ellipse.

21–24 Find the mass and center of gravity of the lamina with density δ.

21. A lamina bounded by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$; $\delta = 2$.

22. A lamina bounded by the graph of $x = y^4$ and the line $x = 1$; $\delta = 15$.

23. A lamina bounded by the graph of $y = |x|$ and the line $y = 1$; $\delta = 3$.

24. A lamina bounded by the x-axis and the graph of the equation $y = 1 - x^2$; $\delta = 3$.

C 25–26 Use a CAS to find the mass and center of gravity of the lamina with density δ.

25. A lamina bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$; $\delta = 4$.

26. A lamina bounded by the graphs of $y = \cos x$, $y = \sin x$, $x = 0$, and $x = \pi/4$; $\delta = 1 + \sqrt{2}$.

27–30 True–False Determine whether the statement is true or false. Explain your answer. [In Exercise 30, assume that the (rotated) square lies in the xy-plane to the right of the y-axis.]

27. The centroid of a rectangle is the intersection of the diagonals of the rectangle.

28. The centroid of a rhombus is the intersection of the diagonals of the rhombus.

29. The centroid of an equilateral triangle is the intersection of the medians of the triangle.

30. By rotating a square about its center, it is possible to change the volume of the solid of revolution generated by revolving the square about the y-axis.