Assignment 3 MA1123 Due Friday 23rd.

1. From the definition derive the derivative of the cube root of x.

2. From the definition derive the derivative of cos x

3. Find $\frac{dy}{dx}$

   (a) $y = \sec x$

   (b) $y = \sin\sqrt{\cos(x^3 + 2x + 1)}$

   (c) $x^2y^3 + \cos(xy) = xy$

   (d) $y = x^2 \cos x \sin x$

4. Find

   (a) $\lim_{x\to\infty} \frac{3x^3+2x+1}{4x^3+2x^2+2}$

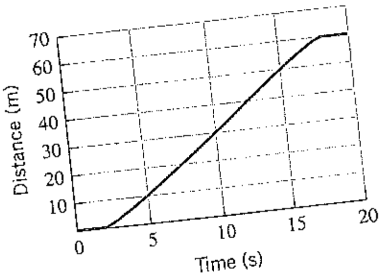
   (b) $\lim_{x\to\infty} \frac{3x^3+2x+1}{4x^4+2x^2-7}$

   (c) $\lim_{x\to 0} x \sin\frac{1}{x}$

   (d) $\lim_{x\to 0} \frac{\sin 5x}{\sin 7x}$

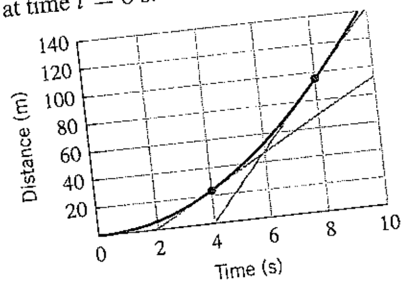
5. On page 120 of the text, attached, 5,6, and 19.

6. On page 173, attached, 12 and 20.



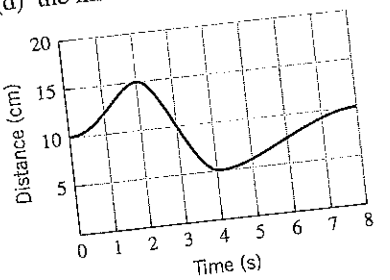
◄ Figure Ex-1

2. The accompanying figure shows the position versus time curve for an automobile over a period of time of 10 s. Use the line segments shown in the figure to estimate the instantaneous velocity of the automobile at time $t = 4$ s and again at time $t = 8$ s.



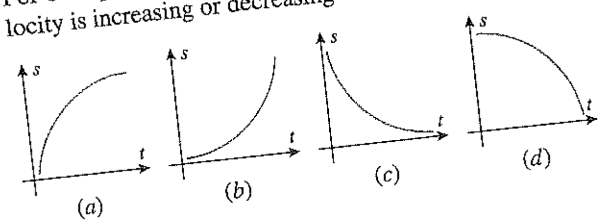
◄ Figure Ex-2

3. The accompanying figure shows the position versus time curve for a certain particle moving along a straight line. Estimate each of the following from the graph:
   (a) the average velocity over the interval $0 \le t \le 3$
   (b) the values of t at which the instantaneous velocity is zero
   (c) the values of t at which the instantaneous velocity is either a maximum or a minimum
   (d) the instantaneous velocity when $t = 3$ s.



◄ Figure Ex-3

4. The accompanying figure shows the position versus time curves of four different particles moving on a straight line. For each particle, determine whether its instantaneous velocity is increasing or decreasing with time.



▲ Figure Ex-4

FOCUS ON CONCEPTS

5. If a particle moves at constant velocity, what can you say about its position versus time curve?

6. An automobile, initially at rest, begins to move along a straight track. The velocity increases steadily until suddenly the driver sees a concrete barrier in the road and applies the brakes sharply at time t_0. The car decelerates rapidly, but it is too late—the car crashes into the barrier at time t_1 and instantaneously comes to rest. Sketch a position versus time curve that might represent the motion of the car. Indicate how characteristics of your curve correspond to the events of this scenario.

7–10 For each exercise, sketch a curve and a line L satisfying the stated conditions.

7. L is tangent to the curve and intersects the curve in at least two points.

8. L intersects the curve in exactly one point, but L is not tangent to the curve.

9. L is tangent to the curve at two different points.

10. L is tangent to the curve at two different points and intersects the curve at a third point.

11–14 A function $y = f(x)$ and values of x_0 and x_1 are given.
(a) Find the average rate of change of y with respect to x over the interval $[x_0, x_1]$.
(b) Find the instantaneous rate of change of y with respect to x at the specified value of x_0.
(c) Find the instantaneous rate of change of y with respect to x at an arbitrary value of x_0.
(d) The average rate of change in part (a) is the slope of a certain secant line, and the instantaneous rate of change in part (b) is the slope of a certain tangent line. Sketch the graph of $y = f(x)$ together with those two lines.

11. $y = 2x^2$; $x_0 = 0$, $x_1 = 1$ 12. $y = x^3$; $x_0 = 1$, $x_1 = 2$
13. $y = 1/x$; $x_0 = 2$, $x_1 = 3$ 14. $y = 1/x^2$; $x_0 = 1$, $x_1 = 2$

15–18 A function $y = f(x)$ and an x-value x_0 are given.
(a) Find a formula for the slope of the tangent line to the graph of f at a general point $x = x_0$.
(b) Use the formula obtained in part (a) to find the slope of the tangent line for the given value of x_0.

15. $f(x) = x^2 - 1$; $x_0 = -1$
16. $f(x) = x^2 + 3x + 2$; $x_0 = 2$
17. $f(x) = x + \sqrt{x}$; $x_0 = 1$
18. $f(x) = 1/\sqrt{x}$; $x_0 = 4$

19–22 True–False Determine whether the statement is true or false. Explain your answer.

19. If $\lim_{x \to 1} \dfrac{f(x) - f(1)}{x - 1} = 3$, then $\lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = 3$

(c) Use the equation in part (b) to find an equation that relates dA/dt and dx/dt.
(d) At a certain instant the sides are 3 ft long and increasing at a rate of 2 ft/min. How fast is the area increasing at that instant?

6. In parts (a)–(d), let A be the area of a circle of radius r, and assume that r increases with the time t.
(a) Draw a picture of the circle with the labels A and r placed appropriately.
(b) Write an equation that relates A and r.
(c) Use the equation in part (b) to find an equation that relates dA/dt and dr/dt.
(d) At a certain instant the radius is 5 cm and increasing at the rate of 2 cm/s. How fast is the area increasing at that instant?

7. Let V be the volume of a cylinder having height h and radius r, and assume that h and r vary with time.
(a) How are dV/dt, dh/dt, and dr/dt related?
(b) At a certain instant, the height is 6 in and increasing at 1 in/s, while the radius is 10 in and decreasing at 1 in/s. How fast is the volume changing at that instant? Is the volume increasing or decreasing at that instant?

8. Let l be the length of a diagonal of a rectangle whose sides have lengths x and y, and assume that x and y vary with time.
(a) How are dl/dt, dx/dt, and dy/dt related?
(b) If x increases at a constant rate of $\frac{1}{2}$ ft/s and y decreases at a constant rate of $\frac{1}{4}$ ft/s, how fast is the size of the diagonal changing when $x = 3$ ft and $y = 4$ ft? Is the diagonal increasing or decreasing at that instant?

9. Let θ (in radians) be an acute angle in a right triangle, and let x and y, respectively, be the lengths of the sides adjacent to and opposite θ. Suppose also that x and y vary with time.
(a) How are $d\theta/dt$, dx/dt, and dy/dt related?
(b) At a certain instant, $x = 2$ units and is increasing at 1 unit/s, while $y = 2$ units and is decreasing at $\frac{1}{4}$ unit/s. How fast is θ changing at that instant? Is θ increasing or decreasing at that instant?

10. Suppose that $z = x^3y^2$, where both x and y are changing with time. At a certain instant when $x = 1$ and $y = 2$, x is decreasing at the rate of 2 units/s, and y is increasing at the rate of 3 units/s. How fast is z changing at this instant? Is z increasing or decreasing?

11. The minute hand of a certain clock is 4 in long. Starting from the moment when the hand is pointing straight up, how fast is the area of the sector that is swept out by the hand increasing at any instant during the next revolution of the hand?

12. A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/s. How rapidly is the area enclosed by the ripple increasing at the end of 10 s?

13. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/h. How fast is the radius of the spill increasing when the area is 9 mi^2?

14. A spherical balloon is inflated so that its volume is increasing at the rate of 3 ft^3/min. How fast is the diameter of the balloon increasing when the radius is 1 ft?

15. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?

16. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

17. A 13 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot be moving away from the wall when the top is 5 ft above the ground?

18. A 10 ft plank is leaning against a wall. If at a certain instant the bottom of the plank is 2 ft from the wall and is being pushed toward the wall at the rate of 6 in/s, how fast is the acute angle that the plank makes with the ground increasing?

19. A softball diamond is a square whose sides are 60 ft long. Suppose that a player running from first to second base has a speed of 25 ft/s at the instant when she is 10 ft from second base. At what rate is the player's distance from home plate changing at that instant?

20. A rocket, rising vertically, is tracked by a radar station that is on the ground 5 mi from the launchpad. How fast is the rocket rising when it is 4 mi high and its distance from the radar station is increasing at a rate of 2000 mi/h?

21. For the camera and rocket shown in Figure 2.8.5, at what rate is the camera-to-rocket distance changing when the rocket is 4000 ft up and rising vertically at 880 ft/s?

22. For the camera and rocket shown in Figure 2.8.5, at what rate is the rocket rising when the elevation angle is $\pi/4$ radians and increasing at a rate of 0.2 rad/s?

23. A satellite is in an elliptical orbit around the Earth. Its distance r (in miles) from the center of the Earth is given by

$$r = \frac{4995}{1 + 0.12\cos\theta}$$

where θ is the angle measured from the point on the orbit nearest the Earth's surface (see the accompanying figure on the next page).
(a) Find the altitude of the satellite at ***perigee*** (the point nearest the surface of the Earth) and at ***apogee*** (the point farthest from the surface of the Earth). Use 3960 mi as the radius of the Earth.
(b) At the instant when θ is 120°, the angle θ is increasing at the rate of 2.7°/min. Find the altitude of the