Assignment 11 MA1123

Christmas 2013.

Q 1. Prove that $\lim_{n\to\infty} a_n = L$ for a sequence $\{a_n\} \iff \lim_{n_k\to\infty} a_{n_k} = L$ for all subsequences $\{a_{n_k}\}$.

Q 2. Prove that if $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} b_n = L_2 \neq 0$, then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L_1}{L_2}$$

Q 3. Prove l.u.b. S = g.l.b. $-S$ for all $S \subset \mathbb{R}$ Note $-S = \{-a : a \in S\}$.

Q 4. Find the limit of the following series, if it exists

$$\sum_{k=1}^{\infty} \frac{1}{9k^2+3k-2}$$

Q 5. Determine if the following series converge or diverge

(i) $$\sum_{k=1}^{\infty} \frac{k^2+2k+1}{k^3+k^2-1}$$

(ii) $$\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$$

(iii) $$\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$$

Q 6. Find the Taylor Series about $x=0$ for, e^x, $\cos x$, $\sin x$.

Q 7. Find the remainder $R_n(x)$ for e^x and show that $R_n(x) \to 0$ for all x.

Q 8. Determine where $$\sum_{k=1}^{\infty} (-1)^k \frac{(x-4)^k}{(k+1)^2}$$

converges absolutely, conditionally, or diverges.