MA1123 Assignment8

[due Monday 24 November, 2014]

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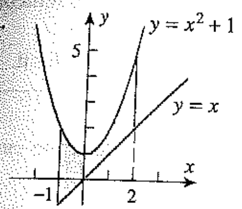
These pages follow.

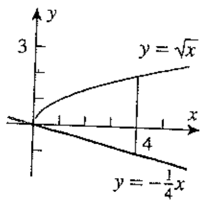
✓QUICK CHECK EXERCISES 5.1 (See page 355 for answers.)

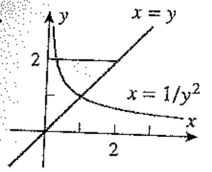
1. An integral expression for the area of the region between the curves $y = 20 - 3x^2$ and $y = 3\sqrt{x}$ and bounded on the sides by $x = 0$ and $x = 2$ is ______.
2. An integral expression for the area of the parallelogram bounded by $y = 2x + 8$, $y = 2x - 3$, $x = -1$, and $x = 5$ is ______. The value of this integral is ______.
3. (a) The points of intersection for the circle $x^2 + y^2 = 4$ and the line $y = x + 2$ are ______ and ______.
   (b) Expressed as a definite integral with respect to x, ______ gives the area of the region inside the circle $x^2 + y^2 = 4$ and above the line $y = x + 2$.
   (c) Expressed as a definite integral with respect to y, ______ gives the area of the region described in part (b).
4. The area of the region enclosed by the curves $y = x^2$ and $y = \sqrt[3]{x}$ is ______.

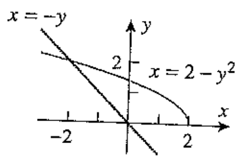
EXERCISE SET 5.1 Graphing Utility C CAS

1–4 Find the area of the shaded region.

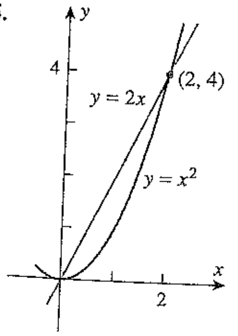
1. 

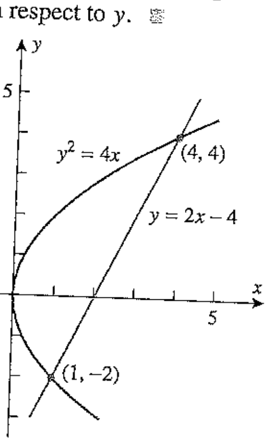
2. 

3. 

4. 

5–6 Find the area of the shaded region by (a) integrating with respect to x and (b) integrating with respect to y.

5. 

6. 

-14 Sketch the region enclosed by the curves and find its area.

- $y = x^2,\ y = \sqrt{x},\ x = \frac{1}{4},\ x = 1$
- $y = x^3 - 4x,\ y = 0,\ x = 0,\ x = 2$
- $y = \cos 2x,\ y = 0,\ x = \pi/4,\ x = \pi/2$
- $y = \sec^2 x,\ y = 2,\ x = -\pi/4,\ x = \pi/4$
- $x = \sin y,\ x = 0,\ y = \pi/4,\ y = 3\pi/4$
- $x^2 = y,\ x = y - 2$

13. $y = 2 + |x - 1|,\ y = -\frac{1}{5}x + 7$
14. $y = x,\ y = 4x,\ y = -x + 2$

15–20 Use a graphing utility, where helpful, to find the area of the region enclosed by the curves.

15. $y = x^3 - 4x^2 + 3x,\ y = 0$
16. $y = x^3 - 2x^2,\ y = 2x^2 - 3x$
17. $y = \sin x,\ y = \cos x,\ x = 0,\ x = 2\pi$
18. $y = x^3 - 4x,\ y = 0$
19. $x = y^3 - y,\ x = 0$
20. $x = y^3 - 4y^2 + 3y,\ x = y^2 - y$

21–24 True–False Determine whether the statement is true or false. Explain your answer. [In each exercise, assume that f and g are distinct continuous functions on $[a, b]$ and that A denotes the area of the region bounded by the graphs of $y = f(x)$, $y = g(x)$, $x = a$, and $x = b$.]

21. If f and g differ by a positive constant c, then $A = c(b - a)$.
22. If
$$\int_a^b [f(x) - g(x)]\, dx = -3$$
then $A = 3$.
23. If
$$\int_a^b [f(x) - g(x)]\, dx = 0$$
then the graphs of $y = f(x)$ and $y = g(x)$ cross at least once on $[a, b]$.
24. If
$$A = \left| \int_a^b [f(x) - g(x)]\, dx \right|$$
then the graphs of $y = f(x)$ and $y = g(x)$ don't cross on $[a, b]$.
25. C Use a CAS to find the area enclosed by $y = 3 - 2x$ and $y = x^6 + 2x^5 - 3x^4 + x^2$.
26. C Use a CAS to find the exact area enclosed by the curves $y = x^5 - 2x^3 - 3x$ and $y = x^3$.
27. Find a horizontal line $y = k$ that divides the area between $y = x^2$ and $y = 9$ into two equal parts.
28. Find a vertical line $x = k$ that divides the area enclosed by $x = \sqrt{y}$, $x = 2$, and $y = 0$ into two equal parts.

29. (a) Find the area of the region enclosed by the parabola $y = 2x - x^2$ and the x-axis.
(b) Find the value of m so that the line $y = mx$ divides the region in part (a) into two regions of equal area.

30. Find the area between the curve $y = \sin x$ and the line segment joining the points $(0, 0)$ and $(5\pi/6, 1/2)$ on the curve.

31–33 Use Newton's Method (Section 3.7), where needed, to approximate the x-coordinates of the intersections of the curves to at least four decimal places, and then use those approximations to approximate the area of the region.

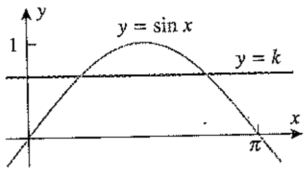
31. The region that lies below the curve $y = \sin x$ and above the line $y = 0.2x$, where $x \geq 0$.

32. The region enclosed by the graphs of $y = x^2$ and $y = \cos x$.

33. The region that is enclosed by the curves $y = x^2 - 1$ and $y = 2\sin x$.

C **34.** Referring to the accompanying figure, use a CAS to estimate the value of k so that the areas of the shaded regions are equal.

Source: This exercise is based on Problem A1 that was posed in the Fifty-Fourth Annual William Lowell Putnam Mathematical Competition.



◀ **Figure Ex-34**

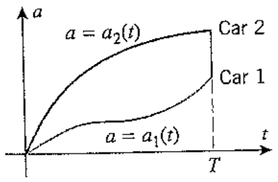
FOCUS ON CONCEPTS

35. Two racers in adjacent lanes move with velocity functions $v_1(t)$ m/s and $v_2(t)$ m/s, respectively. Suppose that the racers are even at time $t = 60$ s. Interpret the value of the integral

$$\int_0^{60} [v_2(t) - v_1(t)]\, dt$$

in this context.

36. The accompanying figure shows acceleration versus time curves for two cars that move along a straight track, accelerating from rest at the starting line. What does the area A between the curves over the interval $0 \leq t \leq T$ represent? Justify your answer.



◀ **Figure Ex-36**

37. Suppose that f and g are integrable on $[a, b]$, but neither $f(x) \geq g(x)$ nor $g(x) \geq f(x)$ holds for all x in $[a, b]$ [i.e., the curves $y = f(x)$ and $y = g(x)$ are intertwined].
(a) What is the geometric significance of the integral

$$\int_a^b [f(x) - g(x)]\, dx?$$

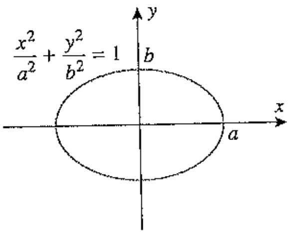
(b) What is the geometric significance of the integral

$$\int_a^b |f(x) - g(x)|\, dx?$$

38. Let $A(n)$ be the area in the first quadrant enclosed by the curves $y = \sqrt[n]{x}$ and $y = x$.
(a) By considering how the graph of $y = \sqrt[n]{x}$ changes as n increases, make a conjecture about the limit of $A(n)$ as $n \to +\infty$.
(b) Confirm your conjecture by calculating the limit.

39. Find the area of the region enclosed between the curve $x^{1/2} + y^{1/2} = a^{1/2}$ and the coordinate axes.

40. Show that the area of the ellipse in the accompanying figure is πab. [*Hint:* Use a formula from geometry.]



◀ **Figure Ex-40**

41. Writing Suppose that f and g are continuous on $[a, b]$ but that the graphs of $y = f(x)$ and $y = g(x)$ cross several times. Describe a step-by-step procedure for determining the area bounded by the graphs of $y = f(x)$, $y = g(x)$, $x = a$, and $x = b$.

42. Writing Suppose that R and S are two regions in the xy-plane that lie between a pair of lines L_1 and L_2 that are parallel to the y-axis. Assume that each line between L_1 and L_2 that is parallel to the y-axis intersects R and S in line segments of equal length. Give an informal argument that the area of R is equal to the area of S. (Make reasonable assumptions about the boundaries of R and S.)

QUICK CHECK EXERCISES 5.2

(See page 365 for answers.)

1. A solid S extends along the x-axis from $x = 1$ to $x = 3$. For x between 1 and 3, the cross-sectional area of S perpendicular to the x-axis is $3x^2$. An integral expression for the volume of S is ______. The value of this integral is ______.

2. A solid S is generated by revolving the region between the x-axis and the curve $y = \sqrt{\sin x}$ $(0 \leq x \leq \pi)$ about the x-axis.
   (a) For x between 0 and π, the cross-sectional area of S perpendicular to the x-axis at x is $A(x) =$ ______.
   (b) An integral expression for the volume of S is ______.
   (c) The value of the integral in part (b) is ______.

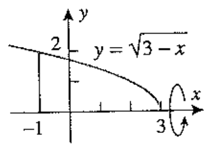
3. A solid S is generated by revolving the region enclosed by the line $y = 2x + 1$ and the curve $y = x^2 + 1$ about the x-axis.
   (a) For x between ______ and ______, the cross sectional area of S perpendicular to the x-axis at x i: $A(x) =$ ______.
   (b) An integral expression for the volume of S is ______

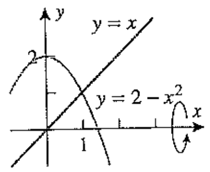
4. A solid S is generated by revolving the region enclosed b the line $y = x + 1$ and the curve $y = x^2 + 1$ about the y axis.
   (a) For y between ______ and ______, the cross sectional area of S perpendicular to the y-axis at y i $A(y) =$ ______.
   (b) An integral expression for the volume of S is ______

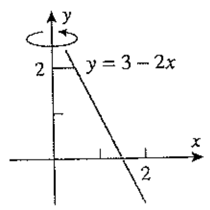
EXERCISE SET 5.2

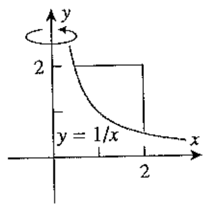
C CAS

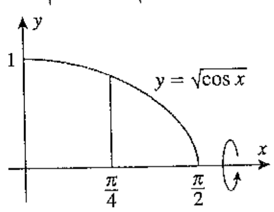
1–8 Find the volume of the solid that results when the shaded region is revolved about the indicated axis.

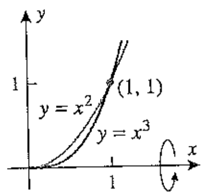
1. 

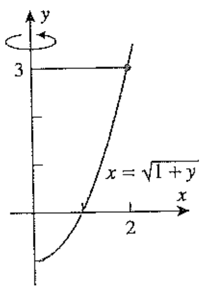
2. 

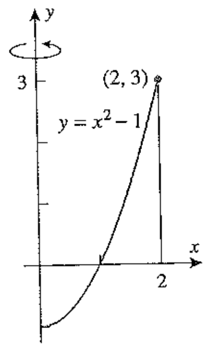
3. 

4. 

5. 

6. 

7. 

8. 

9. Find the volume of the solid whose base is the regio bounded between the curve $y = x^2$ and the x-axis fro $x = 0$ to $x = 2$ and whose cross sections taken perpendic ular to the x-axis are squares.

10. Find the volume of the solid whose base is the regio bounded between the curve $y = \sec x$ and the x-axis fro $x = \pi/4$ to $x = \pi/3$ and whose cross sections taken per pendicular to the x-axis are squares.

11–14 Find the volume of the solid that results when the regio enclosed by the given curves is revolved about the x-axis.

11. $y = \sqrt{25 - x^2}$, $y = 3$

12. $y = 9 - x^2$, $y = 0$

13. $x = \sqrt{y}$, $x = y/4$

14. $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$
[*Hint:* Use the identity $\cos 2x = \cos^2 x - \sin^2 x$.]

15. Find the volume of the solid whose base is the regio bounded between the curve $y = x^3$ and the y-axis fro $y = 0$ to $y = 1$ and whose cross sections taken perpendic ular to the y-axis are squares.

16. Find the volume of the solid whose base is the region en closed between the curve $x = 1 - y^2$ and the y-axis an whose cross sections taken perpendicular to the y-axis ar squares.

17–20 Find the volume of the solid that results when the regio enclosed by the given curves is revolved about the y-axis.

17. $x = \csc y$, $y = \pi/4$, $y = 3\pi/4$, $x = 0$

18. $y = x^2$, $x = y^2$

19. $x = y^2$, $x = y + 2$

20. $x = 1 - y^2$, $x = 2 + y^2$, $y = -1$, $y = 1$

21–24 True–False Determine whether the statement is true or false. Explain your answer. [In these exercises, assume that a solid S of volume V is bounded by two parallel planes perpendicular to the x-axis at $x = a$ and $x = b$ and that for each x in $[a, b]$, $A(x)$ denotes the cross-sectional area of S perpendicular to the x-axis.]

21. If each cross section of S perpendicular to the x-axis is a square, then S is a rectangular parallelepiped (i.e., is box shaped).

22. If each cross section of S is a disk or a washer, then S is a solid of revolution.

23. If x is in centimeters (cm), then $A(x)$ must be a quadratic function of x, since units of $A(x)$ will be square centimeters (cm^2).

24. The average value of $A(x)$ on the interval $[a, b]$ is given by $V/(b - a)$.

25. Find the volume of the solid that results when the region above the x-axis and below the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > 0, b > 0)$$

is revolved about the x-axis.

26. Let V be the volume of the solid that results when the region enclosed by $y = 1/x$, $y = 0$, $x = 2$, and $x = b$ $(0 < b < 2)$ is revolved about the x-axis. Find the value of b for which $V = 3$.

27. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x+1}$, $y = \sqrt{2x}$, and $y = 0$ is revolved about the x-axis. [*Hint:* Split the solid into two parts.]

28. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and $y = 0$ is revolved about the x-axis. [*Hint:* Split the solid into two parts.]

FOCUS ON CONCEPTS

29. Suppose that f is a continuous function on $[a, b]$, and let R be the region between the curve $y = f(x)$ and the line $y = k$ from $x = a$ to $x = b$. Using the method of disks, derive with explanation a formula for the volume of a solid generated by revolving R about the line $y = k$. State and explain additional assumptions, if any, that you need about f for your formula.

30. Suppose that v and w are continuous functions on $[c, d]$, and let R be the region between the curves $x = v(y)$ and $x = w(y)$ from $y = c$ to $y = d$. Using the method of washers, derive with explanation a formula for the volume of a solid generated by revolving R about the line $x = k$. State and explain additional assumptions, if any, that you need about v and w for your formula.

31. Consider the solid generated by revolving the shaded region in Exercise 1 about the line $y = 2$.
   (a) Make a conjecture as to which is larger: the volume of this solid or the volume of the solid in Exercise 1. Explain the basis of your conjecture.
   (b) Check your conjecture by calculating this volume and comparing it to the volume obtained in Exercise 1.

32. Consider the solid generated by revolving the shaded region in Exercise 4 about the line $x = 2.5$.
   (a) Make a conjecture as to which is larger: the volume of this solid or the volume of the solid in Exercise 4. Explain the basis of your conjecture.
   (b) Check your conjecture by expressing the difference in the two volumes as a single definite integral. [*Hint:* Sketch the graph of the integrand.]

33. Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 9$ is revolved about the line $x = 9$.

34. Find the volume of the solid that results when the region in Exercise 33 is revolved about the line $y = 3$.

35. Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

36. Find the volume of the solid that results when the region in Exercise 35 is revolved about the line $x = -1$.

37. Find the volume of the solid that results when the region enclosed by $y = x^2$ and $y = x^3$ is revolved about the line $x = 1$.

38. Find the volume of the solid that results when the region in Exercise 37 is revolved about the line $y = -1$.

39. A nose cone for a space reentry vehicle is designed so that a cross section, taken x ft from the tip and perpendicular to the axis of symmetry, is a circle of radius $\frac{1}{4}x^2$ ft. Find the volume of the nose cone given that its length is 20 ft.

40. A certain solid is 1 ft high, and a horizontal cross section taken x ft above the bottom of the solid is an annulus of inner radius x^2 ft and outer radius $\sqrt{x}$ ft. Find the volume of the solid.

41. Find the volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the x-axis are squares.

42. The base of a certain solid is the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.

43. In parts (a)–(c) find the volume of the solid whose base is enclosed by the circle $x^2 + y^2 = 1$ and whose cross sections taken perpendicular to the x-axis are as indicated. *(cont.)*

26. R is the region in the first quadrant bounded by the graphs of $y = \sqrt{1-x^2}$, $y = 0$, and $x = 0$.

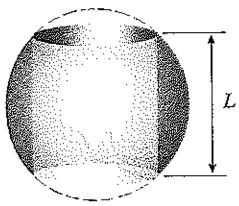
27. Use cylindrical shells to find the volume of the solid that is generated when the region that is enclosed by $y = 1/x^3$, $x = 1$, $x = 2$, $y = 0$ is revolved about the line $x = -1$.

28. Use cylindrical shells to find the volume of the solid that is generated when the region that is enclosed by $y = x^3$, $y = 1$, $x = 0$ is revolved about the line $y = 1$.

29. Use cylindrical shells to find the volume of the cone generated when the triangle with vertices $(0, 0)$, $(0, r)$, $(h, 0)$, where $r > 0$ and $h > 0$, is revolved about the x-axis.

30. The region enclosed between the curve $y^2 = kx$ and the line $x = \frac{1}{4}k$ is revolved about the line $x = \frac{1}{2}k$. Use cylindrical shells to find the volume of the resulting solid. (Assume $k > 0$.)

31. As shown in the accompanying figure, a cylindrical hole is drilled all the way through the center of a sphere. Show that the volume of the remaining solid depends only on the length L of the hole, not on the size of the sphere.



◄ Figure Ex-31

32. Use cylindrical shells to find the volume of the torus obtained by revolving the circle $x^2 + y^2 = a^2$ about the line $x = b$, where $b > a > 0$. [*Hint:* It may help in the integration to think of an integral as an area.]

33. Let V_x and V_y be the volumes of the solids that result when the region enclosed by $y = 1/x$, $y = 0$, $x = \frac{1}{2}$, and $x = b$ $\left(b > \frac{1}{2}\right)$ is revolved about the x-axis and y-axis, respectively. Is there a value of b for which $V_x = V_y$?

34. **Writing** Faced with the problem of computing the volume of a solid of revolution, how would you go about deciding whether to use the method of disks/washers or the method of cylindrical shells?

35. **Writing** With both the method of disks/washers and with the method of cylindrical shells, we integrate an "area" to get the volume of a solid of revolution. However, these two approaches differ in very significant ways. Write a brief paragraph that discusses these differences.

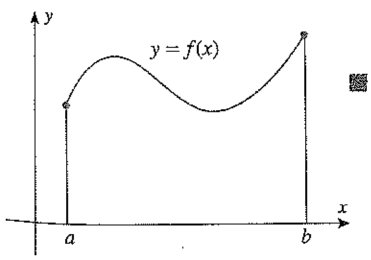
✔ QUICK CHECK ANSWERS 5.3

1. (a) $2\pi x(1+\sqrt{x})$ (b) $\int_1^4 2\pi x(1+\sqrt{x})\,dx$ 2. (a) $2\pi(5-x)(1+\sqrt{x})$ (b) $\int_1^4 2\pi(5-x)(1+\sqrt{x})\,dx$

3. $\int_0^4 2\pi y[4-(y-2)^2]\,dy$

5.4 LENGTH OF A PLANE CURVE

In this section we will use the tools of calculus to study the problem of finding the length of a plane curve.



▲ Figure 5.4.1

ARC LENGTH

Our first objective is to define what we mean by the ***length*** (also called the ***arc length***) of a plane curve $y = f(x)$ over an interval $[a, b]$ (Figure 5.4.1). Once that is done we will be able to focus on the problem of computing arc lengths. To avoid some complications that would otherwise occur, we will impose the requirement that f' be continuous on $[a, b]$, in which case we will say that $y = f(x)$ is a ***smooth curve*** on $[a, b]$ or that f is a ***smooth function*** on $[a, b]$. Thus, we will be concerned with the following problem.

3–8 Find the exact arc length of the curve over the interval.

3. $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$
4. $x = \frac{1}{3}(y^2 + 2)^{3/2}$ from $y = 0$ to $y = 1$
5. $y = x^{2/3}$ from $x = 1$ to $x = 8$
6. $y = (x^6 + 8)/(16x^2)$ from $x = 2$ to $x = 3$
7. $24xy = y^4 + 48$ from $y = 2$ to $y = 4$
8. $x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$ from $y = 1$ to $y = 4$

9–12 True–False Determine whether the statement is true or false. Explain your answer.

9. The graph of $y = \sqrt{1 - x^2}$ is a smooth curve on $[-1, 1]$.
10. The approximation
$$L \approx \sum_{k=1}^{n} \sqrt{(\Delta x_k)^2 + [f(x_k) - f(x_{k-1})]^2}$$
for arc length is not expressed in the form of a Riemann sum.
11. The approximation
$$L \approx \sum_{k=1}^{n} \sqrt{1 + [f'(x_k^*)]^2}\, \Delta x_k$$
for arc length is exact when f is a linear function of x.
12. In our definition of the arc length for the graph of $y = f(x)$, we need $f'(x)$ to be a continuous function in order for f to satisfy the hypotheses of the Mean-Value Theorem (3.8.2).

FOCUS ON CONCEPTS

13. Consider the curve $y = x^{2/3}$.
    (a) Sketch the portion of the curve between $x = -1$ and $x = 8$.
    (b) Explain why Formula (4) cannot be used to find the arc length of the curve sketched in part (a).
    (c) Find the arc length of the curve sketched in part (a).
14. The curve segment $y = x^2$ from $x = 1$ to $x = 2$ may also be expressed as the graph of $x = \sqrt{y}$ from $y = 1$ to $y = 4$. Set up two integrals that give the arc length of this curve segment, one by integrating with respect to x, and the other by integrating with respect to y. Demonstrate a substitution that verifies that these two integrals are equal.
15. Consider the curve segments $y = x^2$ from $x = \frac{1}{2}$ to $x = 2$ and $y = \sqrt{x}$ from $x = \frac{1}{4}$ to $x = 4$.
    (a) Graph the two curve segments and use your graphs to explain why the lengths of these two curve segments should be equal.
    (b) Set up integrals that give the arc lengths of the curve segments by integrating with respect to x. Demonstrate a substitution that verifies that these two integrals are equal.
    (c) Set up integrals that give the arc lengths of the curve segments by integrating with respect to y.
    (d) Approximate the arc length of each curve segment using Formula (2) with $n = 10$ equal subintervals.
    (e) Which of the two approximations in part (d) is more accurate? Explain.
    (f) Use the midpoint approximation with $n = 10$ subintervals to approximate each arc length integral in part (b).
    (g) Use a calculating utility with numerical integration capabilities to approximate the arc length integrals in part (b) to four decimal places.
16. Follow the directions of Exercise 15 for the curve segments $y = x^{8/3}$ from $x = 10^{-3}$ to $x = 1$ and $y = x^{3/8}$ from $x = 10^{-8}$ to $x = 1$.
17. Follow the directions of Exercise 15 for the curve segment $y = 1 + 1/x$ from $x = 1$ to $x = 3$ and for the curve segment $y = 1/(x - 1)$ from $x = 4/3$ to $x = 2$.
18. Let $y = f(x)$ be a smooth curve on the closed interval $[a, b]$. Prove that if m and M are nonnegative numbers such that $m \le |f'(x)| \le M$ for all x in $[a, b]$, then the arc length L of $y = f(x)$ over the interval $[a, b]$ satisfies the inequalities
$$(b - a)\sqrt{1 + m^2} \le L \le (b - a)\sqrt{1 + M^2}$$
19. Use the result of Exercise 18 to show that the arc length L of $y = \sec x$ over the interval $0 \le x \le \pi/3$ satisfies
$$\frac{\pi}{3} \le L \le \frac{\pi}{3}\sqrt{13}$$

C 20. A basketball player makes a successful shot from the free throw line. Suppose that the path of the ball from the moment of release to the moment it enters the hoop is described by
$$y = 2.15 + 2.09x - 0.41x^2, \quad 0 \le x \le 4.6$$
where x is the horizontal distance (in meters) from the point of release, and y is the vertical distance (in meters) above the floor. Use a CAS or a scientific calculator with a numerical integration capability to approximate the distance the ball travels from the moment it is released to the moment it enters the hoop. Round your answer to two decimal places.

C 21. Find a positive value of k (to two decimal places) such that the curve $y = k \sin x$ has an arc length of $L = 5$ units over the interval from $x = 0$ to $x = \pi$. [*Hint:* Find an integral for the arc length L in terms of k, and then use a CAS or a scientific calculator with a numerical integration capability to find integer values of k at which the values of $L - 5$ have opposite signs. Complete the solution by using the Intermediate-Value Theorem (1.5.8) to approximate the value of k to two decimal places.]

C 22. As shown in the accompanying figure on the next page, a horizontal beam with dimensions 2 in × 6 in × 16 ft is fixed at both ends and is subjected to a uniformly distributed load of 120 lb/ft. As a result of the load, the centerline of the beam undergoes a deflection that is described by
$$y = -1.67 \times 10^{-8}(x^4 - 2Lx^3 + L^2x^2)$$

$x = 2$ are $y = 1$ and $y = 4$. Since $x = \sqrt{y}$, we have $dx/dy = 1/(2\sqrt{y})$, and hence from (5) the surface area S is

$$S = \int_1^4 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$
$$= \int_1^4 2\pi \sqrt{y} \sqrt{1 + \left(\frac{1}{2\sqrt{y}}\right)^2}\, dy$$
$$= \pi \int_1^4 \sqrt{4y+1}\, dy$$
$$= \frac{\pi}{4} \int_5^{17} u^{1/2}\, du \qquad \begin{matrix} u = 4y+1 \\ du = 4\,dy \end{matrix}$$
$$= \frac{\pi}{4} \cdot \frac{2}{3} u^{3/2} \Big]_{u=5}^{17} = \frac{\pi}{6}(17^{3/2} - 5^{3/2}) \approx 30.85 \blacktriangleleft$$

✔ QUICK CHECK EXERCISES 5.5 *(See page 382 for answers.)*

1. If f is a smooth, nonnegative function on $[a, b]$, then the surface area S of the surface of revolution generated by revolving the portion of the curve $y = f(x)$ between $x = a$ and $x = b$ about the x-axis is ______.
2. The lateral area of the frustum with slant height $\sqrt{10}$ and base radii $r_1 = 1$ and $r_2 = 2$ is ______.
3. An integral expression for the area of the surface generated by rotating the line segment joining $(3, 1)$ and $(6, 2)$ about the x-axis is ______.
4. An integral expression for the area of the surface generated by rotating the line segment joining $(3, 1)$ and $(6, 2)$ about the y-axis is ______.

EXERCISE SET 5.5 $\boxed{C}$ CAS

1–4 Find the area of the surface generated by revolving the given curve about the x-axis.

1. $y = 7x,\ 0 \le x \le 1$
2. $y = \sqrt{x},\ 1 \le x \le 4$
3. $y = \sqrt{4 - x^2},\ -1 \le x \le 1$
4. $x = \sqrt[3]{y},\ 1 \le y \le 8$

5–8 Find the area of the surface generated by revolving the given curve about the y-axis.

5. $x = 9y + 1,\ 0 \le y \le 2$
6. $x = y^3,\ 0 \le y \le 1$
7. $x = \sqrt{9 - y^2},\ -2 \le y \le 2$
8. $x = 2\sqrt{1 - y},\ -1 \le y \le 0$

$\boxed{C}$ **9–12** Use a CAS to find the exact area of the surface generated by revolving the curve about the stated axis.

9. $y = \sqrt{x} - \frac{1}{3}x^{3/2},\ 1 \le x \le 3$; x-axis
10. $y = \frac{1}{3}x^3 + \frac{1}{4}x^{-1},\ 1 \le x \le 2$; x-axis
11. $8xy^2 = 2y^6 + 1,\ 1 \le y \le 2$; y-axis
12. $x = \sqrt{16 - y},\ 0 \le y \le 15$; y-axis

$\boxed{C}$ **13–14** Use a CAS or a calculating utility with a numerical integration capability to approximate the area of the surface generated by revolving the curve about the stated axis. Round your answer to two decimal places.

13. $y = \sin x,\ 0 \le x \le \pi$; x-axis
14. $x = \tan y,\ 0 \le y \le \pi/4$; y-axis

15–18 True–False Determine whether the statement is true or false. Explain your answer.

15. The lateral surface area S of a right circular cone with height h and base radius r is $S = \pi r \sqrt{r^2 + h^2}$.
16. The lateral surface area of a frustum of slant height l and base radii r_1 and r_2 is equal to the lateral surface area of a right circular cylinder of height l and radius equal to the average of r_1 and r_2.

CHAPTER 5 REVIEW EXERCISES

1. Describe the method of slicing for finding volumes, and use that method to derive an integral formula for finding volumes by the method of disks.

2. State an integral formula for finding a volume by the method of cylindrical shells, and use Riemann sums to derive the formula.

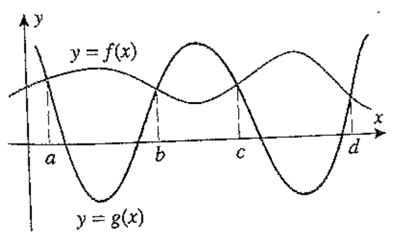
3. State an integral formula for finding the arc length of a smooth curve $y = f(x)$ over an interval $[a, b]$, and use Riemann sums to derive the formula.

4. State an integral formula for the work W done by a variable force $F(x)$ applied in the direction of motion to an object moving from $x = a$ to $x = b$, and use Riemann sums to derive the formula.

5. State an integral formula for the fluid force F exerted on a vertical flat surface immersed in a fluid of weight density ρ, and use Riemann sums to derive the formula.

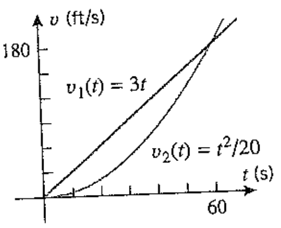
6. Let R be the region in the first quadrant enclosed by $y = x^2$, $y = 2 + x$, and $x = 0$. In each part, set up, but *do not evaluate*, an integral or a sum of integrals that will solve the problem.
   (a) Find the area of R by integrating with respect to x.
   (b) Find the area of R by integrating with respect to y.
   (c) Find the volume of the solid generated by revolving R about the x-axis by integrating with respect to x.
   (d) Find the volume of the solid generated by revolving R about the x-axis by integrating with respect to y.
   (e) Find the volume of the solid generated by revolving R about the y-axis by integrating with respect to x.
   (f) Find the volume of the solid generated by revolving R about the y-axis by integrating with respect to y.
   (g) Find the volume of the solid generated by revolving R about the line $y = -3$ by integrating with respect to x.
   (h) Find the volume of the solid generated by revolving R about the line $x = 5$ by integrating with respect to x.

7. (a) Set up a sum of definite integrals that represents the total shaded area between the curves $y = f(x)$ and $y = g(x)$ in the accompanying figure.
   (b) Find the total area enclosed between $y = x^3$ and $y = x$ over the interval $[-1, 2]$.



◄ Figure Ex-7

8. The accompanying figure shows velocity versus time curves for two cars that move along a straight track, accelerating from rest at a common starting line.
   (a) How far apart are the cars after 60 seconds?
   (b) How far apart are the cars after T seconds, where $0 \le T \le 60$?



◄ Figure Ex-8

9. Let R be the region enclosed by the curves $y = x^2 + 4$, $y = x^3$, and the y-axis. Find and evaluate a definite integral that represents the volume of the solid generated by revolving R about the x-axis.

10. A football has the shape of the solid generated by revolving the region bounded between the x-axis and the parabola $y = 4R(x^2 - \frac{1}{4}L^2)/L^2$ about the x-axis. Find its volume.

11. Find the volume of the solid whose base is the region bounded between the curves $y = \sqrt{x}$ and $y = 1/x$ for $1 \le x \le 4$ and whose cross sections perpendicular to the x-axis are squares.

12. Consider the region enclosed by $y = x^3$, $y = 0$, and $x = 2$. Set up, but *do not evaluate*, an integral that represents the volume of the solid generated by revolving the region about the x-axis using
    (a) disks (b) cylindrical shells.

13. Find the arc length in the second quadrant of the curve $x^{2/3} + y^{2/3} = 4$ from $x = -8$ to $x = -1$.

14. Let C be the curve $y = x^8$ between $x = 1$ and $x = 3$. In each part, set up, but *do not evaluate*, an integral that solves the problem.
    (a) Find the arc length of C by integrating with respect to x.
    (b) Find the arc length of C by integrating with respect to y.

15. Find the area of the surface generated by revolving the curve $y = \sqrt{25 - x}$, $9 \le x \le 16$, about the x-axis.

16. Let C be the curve $27x - y^3 = 0$ between $y = 0$ and $y = 2$. In each part, set up, but *do not evaluate*, an integral or a sum of integrals that solves the problem.
    (a) Find the area of the surface generated by revolving C about the x-axis by integrating with respect to x.
    (b) Find the area of the surface generated by revolving C about the y-axis by integrating with respect to y.
    (c) Find the area of the surface generated by revolving C about the line $y = -2$ by integrating with respect to y.

17. (a) A spring exerts a force of 0.5 N when stretched 0.25 m beyond its natural length. Assuming that Hooke's law applies, how much work was performed in stretching the spring to this length? *(cont.)*