MA1123 Assignment5

[due Tuesday 28 October, 2014]

1. For the following functions find all asymptotes, where they increase or decrease, have local extrema, different concavities, points of inflection, etc. Then draw a rough graph.

   (a) $y = \frac{(x+1)^2}{(x+2)(x+3)}$

   (b) $y = x^{\frac{2}{3}}(\frac{5}{2} - x)$

2. Do 27- 30 from the book, page 215 is atached

3. Do 14,18,40,44,46 from pages 234 and 235, also attached.

15–16 In each part, make a rough sketch of the graph using asymptotes and appropriate limits but no derivatives. Compare your graph to that generated with a graphing utility.

15. (a) $y = \dfrac{3x^2 - 8}{x^2 - 4}$ (b) $y = \dfrac{x^2 + 2x}{x^2 - 1}$

16. (a) $y = \dfrac{2x - x^2}{x^2 + x - 2}$ (b) $y = \dfrac{x^2}{x^2 - x - 2}$

17. Show that $y = x + 3$ is an oblique asymptote of the graph of $f(x) = x^2/(x-3)$. Sketch the graph of $y = f(x)$ showing this asymptotic behavior.

18. Show that $y = 3 - x^2$ is a curvilinear asymptote of the graph of $f(x) = (2 + 3x - x^3)/x$. Sketch the graph of $y = f(x)$ showing this asymptotic behavior.

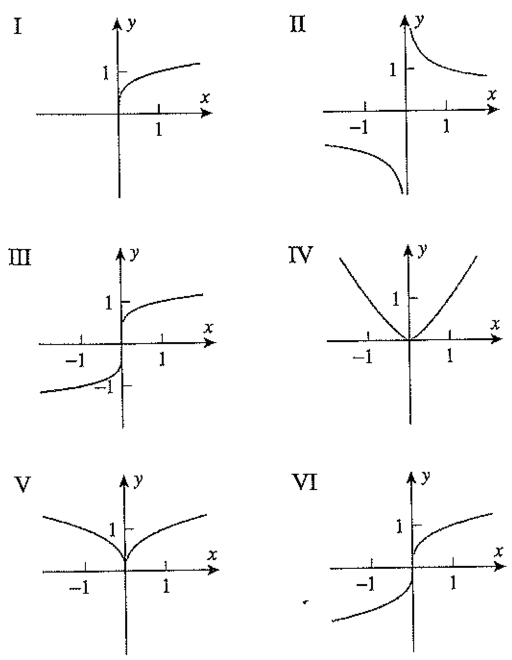
19–24 Sketch a graph of the rational function and label the coordinates of the stationary points and inflection points. Show the horizontal, vertical, oblique, and curvilinear asymptotes and label them with their equations. Label point(s), if any, where the graph crosses an asymptote. Check your work with a graphing utility.

19. $x^2 - \dfrac{1}{x}$ **20.** $\dfrac{x^2 - 2}{x}$ **21.** $\dfrac{(x-2)^3}{x^2}$

22. $x - \dfrac{1}{x} - \dfrac{1}{x^2}$ **23.** $\dfrac{x^3 - 4x - 8}{x + 2}$ **24.** $\dfrac{x^5}{x^2 + 1}$

FOCUS ON CONCEPTS

25. In each part, match the function with graphs I–VI.
(a) $x^{1/3}$ (b) $x^{1/4}$ (c) $x^{1/5}$
(d) $x^{2/5}$ (e) $x^{4/3}$ (f) $x^{-1/3}$



▲ **Figure Ex-25**

26. Sketch the general shape of the graph of $y = x^{1/n}$, and then explain in words what happens to the shape of the graph as n increases if
(a) n is a positive even integer
(b) n is a positive odd integer.

27–30 True–False Determine whether the statement is true or false. Explain your answer.

27. Suppose that $f(x) = P(x)/Q(x)$, where P and Q are polynomials with no common factors. If $y = 5$ is a horizontal asymptote for the graph of f, then P and Q have the same degree.

28. If the graph of f has a vertical asymptote at $x = 1$, then f cannot be continuous at $x = 1$.

29. If the graph of f' has a vertical asymptote at $x = 1$, then f cannot be continuous at $x = 1$.

30. If the graph of f has a cusp at $x = 1$, then f cannot have an inflection point at $x = 1$.

31–38 Give a graph of the function and identify the locations of all critical points and inflection points. Check your work with a graphing utility.

31. $\sqrt{4x^2 - 1}$ **32.** $\sqrt[3]{x^2 - 4}$

33. $2x + 3x^{2/3}$ **34.** $2x^2 - 3x^{4/3}$

35. $4x^{1/3} - x^{4/3}$ **36.** $5x^{2/3} + x^{5/3}$

37. $\dfrac{8 + x}{2 + \sqrt[3]{x}}$ **38.** $\dfrac{8(\sqrt{x} - 1)}{x}$

39–44 Give a graph of the function and identify the locations of all relative extrema and inflection points. Check your work with a graphing utility.

39. $x + \sin x$ **40.** $x - \tan x$

41. $\sqrt{3}\cos x + \sin x$ **42.** $\sin x + \cos x$

43. $\sin^2 x - \cos x$, $\quad -\pi \le x \le 3\pi$

44. $\sqrt{\tan x}$, $\quad 0 \le x < \pi/2$

FOCUS ON CONCEPTS

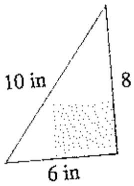
45. The accompanying figure on the next page shows the graph of the *derivative* of a function h that is defined and continuous on the interval $(-\infty, +\infty)$. Assume that the graph of h' has a vertical asymptote at $x = 3$ and that

$$h'(x) \to 0^+ \text{ as } x \to -\infty$$
$$h'(x) \to -\infty \text{ as } x \to +\infty$$

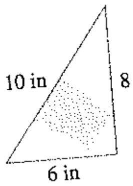
(a) What are the critical points for $h(x)$?
(b) Identify the intervals on which $h(x)$ is increasing.
(c) Identify the x-coordinates of relative extrema for $h(x)$ and classify each as a relative maximum or relative minimum.
(d) Estimate the x-coordinates of inflection points for $h(x)$.

6. A rectangle is to be inscribed in a right triangle having sides of length 6 in, 8 in, and 10 in. Find the dimensions of the rectangle with greatest area assuming the rectangle is positioned as in Figure Ex-6.

7. Solve the problem in Exercise 6 assuming the rectangle is positioned as in Figure Ex-7.



▲ Figure Ex-6



▲ Figure Ex-7

8. A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

9. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10.

10. Find the point P in the first quadrant on the curve $y = x^{-2}$ such that a rectangle with sides on the coordinate axes and a vertex at P has the smallest possible perimeter.

11. A rectangular area of 3200 ft^2 is to be fenced off. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the rectangle of least cost.

12. Show that among all rectangles with perimeter p, the square has the maximum area.

13. Show that among all rectangles with area A, the square has the minimum perimeter.

14. A wire of length 12 in can be bent into a circle, bent into a square, or cut into two pieces to make both a circle and a square. How much wire should be used for the circle if the total area enclosed by the figure(s) is to be
(a) a maximum (b) a minimum?

15. A rectangle R in the plane has corners at $(\pm 8, \pm 12)$, and a 100 by 100 square S is positioned in the plane so that its sides are parallel to the coordinate axes and the lower left corner of S is on the line $y = -3x$. What is the largest possible area of a region in the plane that is contained in both R and S?

16. Solve the problem in Exercise 15 if S is a 16 by 16 square.

17. Solve the problem in Exercise 15 if S is positioned with its lower left corner on the line $y = -6x$.

18. A rectangular page is to contain 42 square inches of printable area. The margins at the top and bottom of the page are each 1 inch, one side margin is 1 inch, and the other side margin is 2 inches. What should the dimensions of the page be so that the least amount of paper is used?

19. A box with a square base is taller than it is wide. In order to send the box through the U.S. mail, the height of the box and the perimeter of the base can sum to no more than 108 in. What is the maximum volume for such a box?

20. A box with a square base is wider than it is tall. In order to send the box through the U.S. mail, the width of the box and the perimeter of one of the (nonsquare) sides of the box can sum to no more than 108 in. What is the maximum volume for such a box?

21. An open box is to be made from a 3 ft by 8 ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume that the box can have.

22. A closed rectangular container with a square base is to have a volume of 2250 in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$3 per in^2. Find the dimensions of the container of least cost.

23. A closed rectangular container with a square base is to have a volume of 2000 cm^3. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of the container of least cost.

24. A container with square base, vertical sides, and open top is to be made from 1000 ft^2 of material. Find the dimensions of the container with greatest volume.

25. A rectangular container with two square sides and an open top is to have a volume of V cubic units. Find the dimensions of the container with minimum surface area.

26. A church window consisting of a rectangle topped by a semicircle is to have a perimeter p. Find the radius of the semicircle if the area of the window is to be maximum.

27. Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a sphere of radius R.

28. Find the dimensions of the right circular cylinder of greates surface area that can be inscribed in a sphere of radius R.

29. A closed, cylindrical can is to have a volume of V cu bic units. Show that the can of minimum surface area achieved when the height is equal to the diameter of th base.

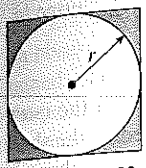
30. A closed cylindrical can is to have a surface area of S squa units. Show that the can of maximum volume is achiev when the height is equal to the diameter of the base.

31. A cylindrical can, open at the top, is to hold 500 cm^3 liquid. Find the height and radius that minimize the amo of material needed to manufacture the can.

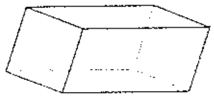
32. A soup can in the shape of a right circular cylinder of ra r and height h is to have a prescribed volume V. The and bottom are cut from squares as shown in Figure E on the next page. If the shaded corners are wasted, but t is no other waste, find the ratio r/h for the can requirin least material (including waste).

33. A box-shaped wire frame consists of two identical squares whose vertices are connected by four straight of equal length (Figure Ex-33 on the next page).

frame is to be made from a wire of length L, what should the dimensions be to obtain a box of greatest volume?



▲ Figure Ex-32

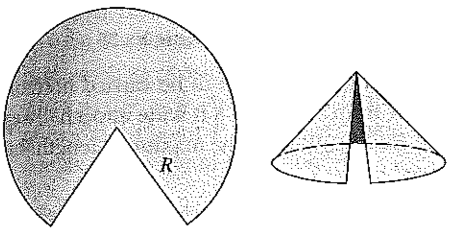


▲ Figure Ex-33

34. Suppose that the sum of the surface areas of a sphere and a cube is a constant.
   (a) Show that the sum of their volumes is smallest when the diameter of the sphere is equal to the length of an edge of the cube.
   (b) When will the sum of their volumes be greatest?

35. Find the height and radius of the cone of slant height L whose volume is as large as possible.

36. A cone is made from a circular sheet of radius R by cutting out a sector and gluing the cut edges of the remaining piece together (Figure Ex-36). What is the maximum volume attainable for the cone?



▲ Figure Ex-36

37. A cone-shaped paper drinking cup is to hold 100 cm^3 of water. Find the height and radius of the cup that will require the least amount of paper.

38. Find the dimensions of the isosceles triangle of least area that can be circumscribed about a circle of radius R.

39. Find the height and radius of the right circular cone with least volume that can be circumscribed about a sphere of radius R.

40. A commercial cattle ranch currently allows 20 steers per acre of grazing land; on the average its steers weigh 2000 lb at market. Estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 lb for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market weight for its cattle?

41. A company mines low-grade nickel ore. If the company mines x tons of ore, it can sell the ore for $p = 225 - 0.25x$ dollars per ton. Find the revenue and marginal revenue functions. At what level of production would the company obtain the maximum revenue?

42. A fertilizer producer finds that it can sell its product at a price of $p = 300 - 0.1x$ dollars per unit when it produces x units of fertilizer. The total production cost (in dollars) for x units is

$$C(x) = 15{,}000 + 125x + 0.025x^2$$

If the production capacity of the firm is at most 1000 units of fertilizer in a specified time, how many units must be manufactured and sold in that time to maximize the profit?

43. (a) A chemical manufacturer sells sulfuric acid in bulk at a price of \$100 per unit. If the daily total production cost in dollars for x units is

$$C(x) = 100{,}000 + 50x + 0.0025x^2$$

and if the daily production capacity is at most 7000 units, how many units of sulfuric acid must be manufactured and sold daily to maximize the profit?
   (b) Would it benefit the manufacturer to expand the daily production capacity?
   (c) Use marginal analysis to approximate the effect on profit if daily production could be increased from 7000 to 7001 units.

44. A firm determines that x units of its product can be sold daily at p dollars per unit, where

$$x = 1000 - p$$

The cost of producing x units per day is

$$C(x) = 3000 + 20x$$

   (a) Find the revenue function $R(x)$.
   (b) Find the profit function $P(x)$.
   (c) Assuming that the production capacity is at most 500 units per day, determine how many units the company must produce and sell each day to maximize the profit.
   (d) Find the maximum profit.
   (e) What price per unit must be charged to obtain the maximum profit?

45. In a certain chemical manufacturing process, the daily weight y of defective chemical output depends on the total weight x of all output according to the empirical formula

$$y = 0.01x + 0.00003x^2$$

where x and y are in pounds. If the profit is \$100 per pound of nondefective chemical produced and the loss is \$20 per pound of defective chemical produced, how many pounds of chemical should be produced daily to maximize the total daily profit?

46. An independent truck driver charges a client \$15 for each hour of driving, plus the cost of fuel. At highway speeds of v miles per hour, the trucker's rig gets $10 - 0.07v$ miles per gallon of diesel fuel. If diesel fuel costs \$2.50 per gallon, what speed v will minimize the cost to the client?

47. A trapezoid is inscribed in a semicircle of radius 2 so that one side is along the diameter (Figure Ex-47 on the next page). Find the maximum possible area for the trapezoid. [*Hint:* Express the area of the trapezoid in terms of θ.]

48. A drainage channel is to be made so that its cross section is a trapezoid with equally sloping sides (Figure Ex-48 on the next page). If the sides and bottom all have a length of 5 ft,