Mathematics 1123

Sample Test

January '12

Answer all questions

1 (i) Define what is meant by a function f from a set X to a set Y.

(ii) Define what it means for $f: X \to Y$ to be onto (surjective).

(iii) What does the vertical line test of a graph tell?

(iv) Define $\lim_{x \to a^-} f(x) = L$, $\lim_{x \to a} f(x) = L$

(v) If $\lim_{x \to a^-} f(x) = L_1$ and $\lim_{x \to a^+} f(x) = L$ prove $L_1 \neq L_2 \Rightarrow \lim_{x \to a} f(x)$ does not exist.

2 (i) Let $f(x) = -x \quad x \leq 0$

$= x^2 \quad x \geq 0.$

Find $f'(0)$ or show it does not exist.

(ii) Let $f(x) = x \sin \frac{1}{x} \quad x \neq 0$

$= 0 \quad x = 0.$

Find $f'(0)$ or show it does not exist.

Is $f(x)$ continuous at $x = 0$?

(iii) If $\lim_{x \to a} f(x) = L_1$, and $\lim_{x \to a} g(x) = L_2$

prove $\lim_{x \to a} f(x).g(x) = L_1 L_2$.

(iv) Find the linear approximation and then the quadratic approximation to $\sin(.05)$

3 (i) Find $\frac{dy}{dx}$ if (a) $y = xe^x \sin x$

(b) $y = \ln(\sin(x^2+1)^3)$

(c) $x^2 y + y x^2 = 1.$

(d) $x = t^2 + 1, \; y = 2t + 1.$

(ii) Let $f(x) = \frac{2(x^2-9)}{x^2-4}$, find where $f(x)$ increases, decreases, is concave up, concave down, local extrema, pts of inflection. Hence sketch this function.

(4) (i) State and prove Rolle's Theorem

(ii) State the Mean Value Theorem and prove that if $f'(x) = 0$, all x in $[a, b]$, then $f(x) =$ Constant on $[a, b]$.

(iii) A Boat B is 1 miles from the shore. The person in the boat wants to get to a point that is 1 mile in land from a point three miles down the shore. If the person can row at 2 miles per hour

and walk at 4 miles per hour what route should they take?

~~(iv) Do~~

5 (i) Use Riemann sums to derive the formula for the arc length of $y = f(x)$ from $x = a$ to $x = b$.

(ii) If $f(x) = 1$ x rational
$= 0$ x irrational

what is $\int_a^b f(x)\,dx$? Explain your reasoning.

(iii) Integrate (a) $\int x \ln x^2\,dx$

(b) $\int x^2 \ln x\,dx$

(c) $\int \frac{x^2+1}{x+2}\,dx$

(d) $\int \frac{x+2}{x^2+1}\,dx$

(e) $\int \frac{x+2}{(x-1)^2(x-2)}\,dx$

(f) $\int \cos^2 x \sin^2 x\,dx$

6 (i) Prove that the volume of a pyramid with square base is $\frac{1}{3} hB$, where h is the height and B is the area of the base.

(ii) Find the volume of the solid formed by revolving the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$, about the y-axis. Do it first by disks and then by cylindrical shells.

(iii) Define hyperbolic sine and cosine. Why are they called hyperbolic?