MA2331 Sample Questions

November 8th 2012

Real Fourier Series
Find the Real Fourier Series of the following functions:

1.

$$f(x) = e^{|x|} \tag{1}$$
$$f(x) = f(x + 2\pi) \tag{2}$$

Answer:

$$a_0 = \frac{2(e^{\pi} - 1)}{\pi} \tag{3}$$
$$a_n = \frac{2(e^{\pi}(-1)^n - 1)}{\pi(n^2 + 1)} \tag{4}$$
$$b_n = 0 \tag{5}$$

2.

$$f(x) = max(\cos(x), 0) \tag{6}$$

Answer:

$$a_0 = \frac{2}{\pi} \tag{7}$$
$$a_n = \frac{2\cos\left(\frac{\pi n}{2}\right)}{\pi(1 - n^2)} \tag{8}$$
$$b_n = 0 \tag{9}$$

Note that the formula for the a_n coefficients does not work with $n = 1$, which must be caculated seperately:

$$a_1 = \frac{1}{2} \tag{10}$$

3.

$$f(x) = x^3 \tag{11}$$
$$f(x) = f(x + 1) \tag{12}$$

Answer:

$$a_0 = 0 \tag{13}$$
$$a_n = 0 \tag{14}$$
$$b_n = \frac{(-1)^n(6-\pi^2n^2)}{4\pi^3n^3} \tag{15}$$

4.

$$f(x) = \sin(ex) \tag{16}$$
$$f(x) = f(x+3) \tag{17}$$

Answer:

$$a_0 = 0 \tag{18}$$
$$a_n = 0 \tag{19}$$
$$b_n = \frac{(-1)^n(8\pi n \sin\left(\frac{3e}{2}\right))}{9e^2-4\pi^2n^2} \tag{20}$$

Complex Fourier Series
Find the complex Fourier Series of

1.

$$f(x) = x + x^2 \tag{21}$$
$$f(x) = f(x+2\pi) \tag{22}$$

Answer:

$$c_n = \frac{2(2+in)(-1)^n}{n^3} \tag{23}$$

This formula will fail for c_0 which must be computed seperately:

$$c_0 = \frac{2\pi^2}{3} \tag{24}$$

2.

$$f(x) = x^3 \tag{25}$$
$$f(x) = f(x+2\pi) \tag{26}$$

Answer:

$$c_n = \frac{(-1)^n i(\pi^2n^2-6)}{n^3} \tag{27}$$

Again this formula will not work for c_0.

$$c_0 = 0 \tag{28}$$

Fourier Transform
Find the Fourier Transform of

1.

$$f(x) = e^{2x} \quad -3 < x < 3 \tag{29}$$
$$= 0 \quad |x| > 3 \tag{30}$$

Answer:

$$\frac{2i\sinh(6-3ik)}{(k+2i)} \tag{31}$$

2.

$$f(x) = \sin(5x) \quad -4 < x < 4 \tag{32}$$
$$= 0 \quad |x| > 4 \tag{33}$$

Answer:

$$\frac{2i\left(k\sin(20)\cos(4k) - 5\cos(20)\sin(4k)\right)}{k^2-25} \tag{34}$$

Find the values of the following Dirac Delta integrals:

1.

$$\int \delta(x-4)(x^3-4x^2-3x+4)dx \tag{35}$$

Answer: 8

2.

$$\int \delta(-8x)(\cos(x-2))dx \tag{36}$$

Answer: $\frac{\cos(-2)}{8} = \frac{\cos(2)}{8}$

3.

$$\int \delta'(x-3)(e^{x^2})dx \tag{37}$$

Answer: $6e^9$

4.

$$\int \delta((x-2)^3)(x^2-4)dx \tag{38}$$

Answer: 0