Determining the Velocity of Light using a Laser Motion Sensor

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1 Introduction and Theory

The velocity of light in a particular medium, c_n, is given by

$$c_n = \frac{c}{n}, \tag{1}$$

where c is the velocity of light in a vacuum and n is the refractive index of that particular medium. The refractive index of a medium can be found as follows: rearranging (1) yields

$$n = \frac{c}{c_n}. \tag{2}$$

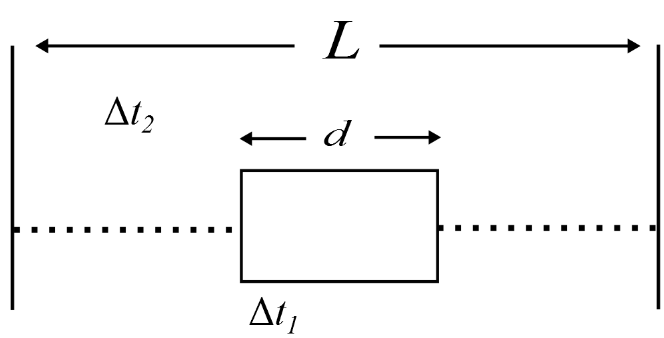

Figure 1: The geomtric setup for measuring the velocity of light in an acrylic block.

In the system shown in the diagram, A light source is emitted, travels through a medium of thickness d, hits a reflector at length L, and travels back through the medium to the source. If the time spent travelling by the light is Δt_1 in glass and Δt_2 in air then the total time is

$$\Delta t = \Delta t_1 + \Delta t_2. \tag{3}$$

Thus, the following expressions are obtained for the velocity of light in the different media:

$$c_n = \frac{2d}{\Delta t_1} = \frac{2d}{\Delta t - \Delta t_2}, \qquad c = \frac{2\,(L-d)}{\Delta t_2}. \tag{4}$$

Substituting these into (2) yields

$$n = 1 + \frac{c}{2d}\Delta t - \frac{L}{d}, \tag{5}$$

and by calibrating the sensor so that $\Delta t = 0$ corresponds to a distance of L, then L is effectively 0 and the final expression for the refractive index is obtained:

$$n = 1 + \frac{c}{2d}\Delta t, \tag{6}$$

meaning the refractive index can be measured using the speed of light in a vacuum, the thickness of the medium and the total time interval.

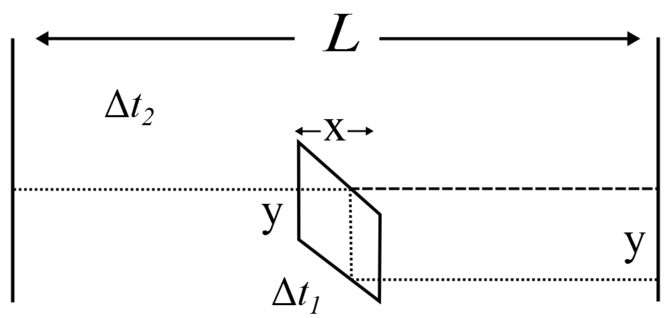

Figure 2: The geomtric setup for measuring the velocity of light in an acrylic prism.

A similar formula can be derived in the case a 45° prism. The geometry of the system is shown in the diagram. Similar expressions can be found for the speed of light in both media:

$$c_n = \frac{2\,(x+y)}{\Delta t_1} = \frac{2\,(x+y)}{\Delta t - \Delta t_2}, \qquad c = \frac{2\,(L-x)}{\Delta t_2}. \tag{7}$$

since it is a 45° prism, then $x = y$, and since L is again effectively 0, the formula becomes

$$n = \frac{1}{2} + \frac{c}{4x}\Delta t. \tag{8}$$

2 Experimental Method

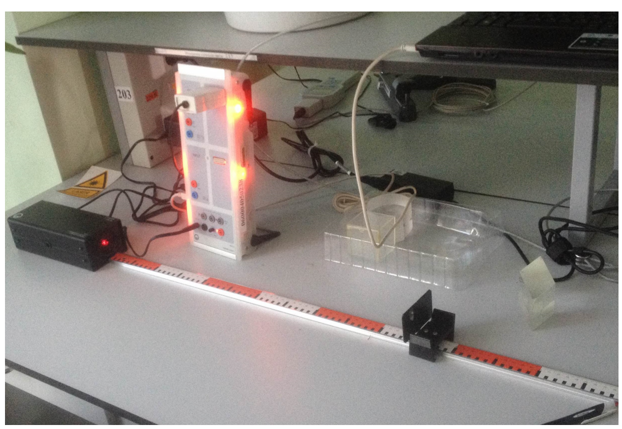

Figure 3: The experimental setup for determining the velocity of light.

In part one of the experiment we determined the velocity of light in air. This was achieved by setting up the apparatus as shown in the diagram, with the laser motion sensor flat on the table and connected to the input of CASSY which was in turn connected to the PC, a metal ruler laid down on the table and a movable buffer with reflective foil set to move back and forth along the ruler. The time interval Δt (the time taken for the light to travel to the buffer, be reflected and once again reach the laser motion sensor) was measured directly using CASSY LAB 2. This calibrated to be 0 for a distance of 50cm. The buffer was moved back in 10cm intervals from the laser and a series of transmission times were recorded. The slope of the plot of Δt vs d, the distance travelled, then gave us our value for the speed of light in air (the length measured on the ruler was doubled because the light had to travel there and back).

In part two of the experiment we found the refractive indices of water and acrylic glass. The position of the buffer was held fixed at a distance of 50cm from the laster and the time interval was calibrated again. The acrylic block was then inserted into the path of the laser beam and positioned at an angle of roughly 3° so that the light reflected from the surface of the block did not travel directly back to the sensor and interfere with our results. Using CASSY, a new Δt was recorded. Finally the length of the block was measured and using (6), the refractive index was calculated.

The acrylic block was removed and an empty glass cell was inserted into the path of the beam, also tilted at an angle of roughly 3° with respect to the laser. The time interval was calibrated and the cell was then filled with water. A value for Δt was recorded using CASSY and the length of the inside of the cell was measured. The refractive index was found, again using (6).

In part three of the experiment we determined the refractive index of a glass prism. The experimental set up was the same as before, with the buffer situated at a fixed distance from the laser. With no prism in the path of the laser, the time interval was calibrated. The prism was placed in the path of the laser beam with the front face perpendicular to the laser beam. The ruler was placed behind the buffer, perpendicular to the laser beam and the buffer was moved laterally along the ruler so that the laser beam again reflects from the surface of the buffer, and Δt was again measured. The dimensions of the prism were noted and using (8), the refractive index of the glass prism was calculated.

3 Results and Analysis

Δt(ns) $\pm$ 0.001ns	d(m) $\pm$ 0.002m
0.000	0.000
0.662	0.200
1.305	0.400
1.972	0.600
2.643	0.800
3.338	1.000
4.020	1.200

A table of values of different time intervals for different distances was recorded and is shown below. These values were plotted on a graph and the slope was found to determine the velocity of light: $c = 2.98 \pm 0.02 \mathrm{m\,s^{-1}}$. The time intervals were found for both acrylic and water were found, and using these with the lengths of the objects, values for the refractive indices were obtained.

	Δt(ns) $\pm$ 0.001ns	d(m) $\pm$ 0.002m	n
Acrylic	0.184	0.049	1.58 ± 0.01
Water	0.104	0.054	1.29 ± 0.01

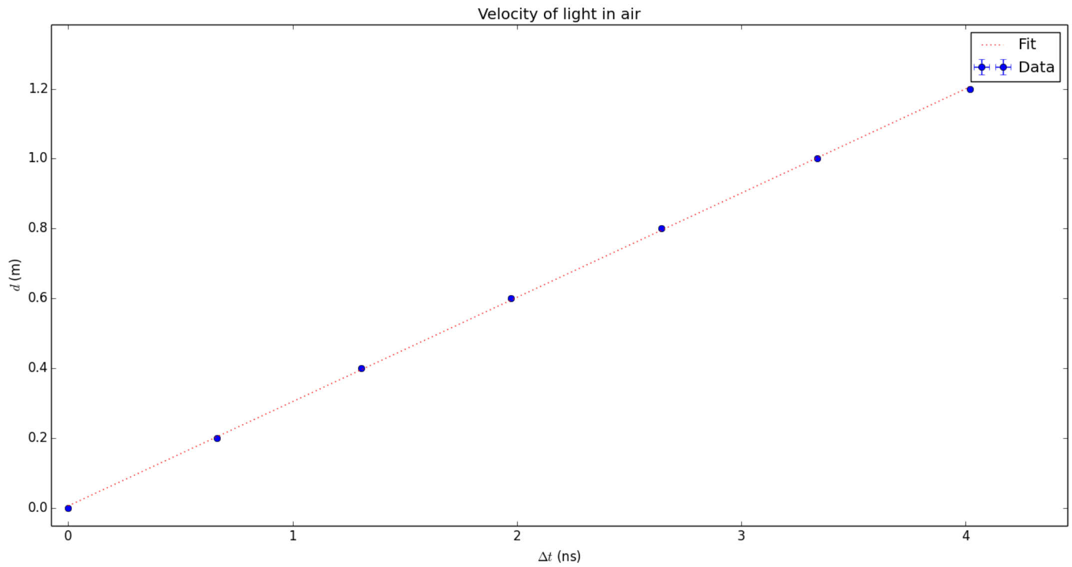

Figure 4: A graph of time intervals vs distance, the slope of which gives the velocity of light in air.

The values obtained for the prism were $\Delta t = 0.772 \pm 0.001$ns, $x = 0.049 \pm 0.001$m. Using the formula obtained for the refractive index of a prism, a value of $n = 1.68 \pm 0.03$ was obtained.

4 Discussion and Conclusions

Our value of the velocity of light in a vacuum was $c = 2.98 \pm 0.02 \mathrm{m\,s^{-1}}$, which is close to the accepted value of $2.997 \mathrm{m\,s^{-1}}$. Our values for the refractive index of acrylic and water were 1.57 ± 0.02 and 1.28 ± 0.02, respectively. Our values are close to the accepted values for these, which are $n = 1.5$ and $n = 1.3$. Using the formula derived for the refractive index of a prism, a value of $n = 1.68 \pm 0.03$ was obtained. Of course, air isn't a perfect vacuum, meaning perfect values can never be obtained. The overall accuracy of the experiment could be improved by using an optical rail, ensuring that all devices are correctly aligned and place. It would also allow for the distances to be more accurately measured.