Investigations of Centered Optical Systems

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1 Introduction and Theory

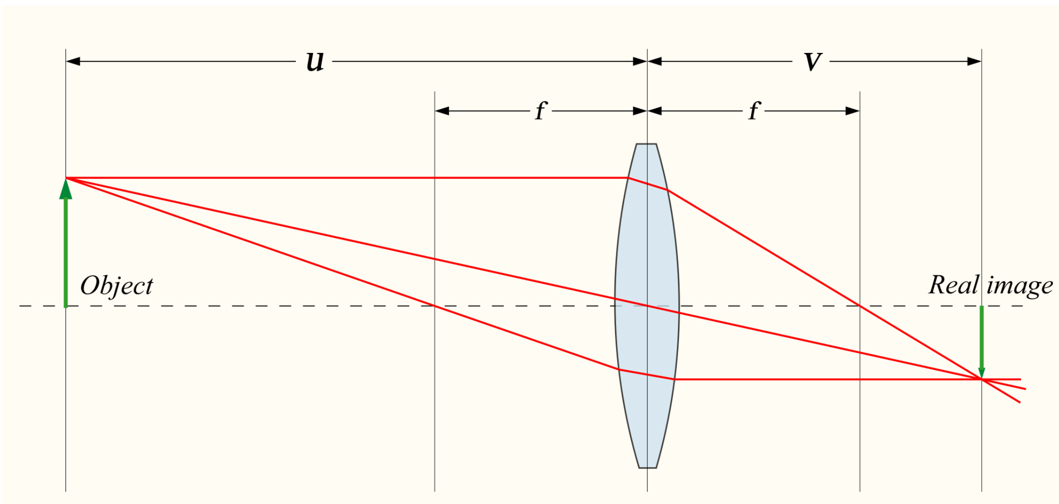

Figure 1: A converging lens.

1.1 Lenses

A lens is an optical device that can be used to focus light and form an image. The centre of the lens is known as the optical centre. Passing through this centre is an axis known as the optic axis. On either side of the lens, there is a focal point lying on the optic axis. Light rays that pass through a converging lens parallel to the optic axis will pass through the focal point. Light that passes through the optical centre will have its direction of propagation undisturbed, and light rays that pass through the focal point before entering the lens will emerge parallel to the optic axis.

In an optical system involving an object, a lens and an image, the object distance, u, between the object and the lens and the image distance, v, between the lens and the image, are related to the focal length, f, by the following equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}. \tag{1}$$

The magnification, m, of an image is the ratio of the size of the image to the size of the object, given by:

$$m = \frac{v}{u}. \tag{2}$$

1.2 Derivation of Bessel's Formula

Consider the setup shown in the diagram. The object and image distances are shown, and $L = u + v$ is the distance from the light source to the screen. Then $v = L - u$. By substituting this into (1), a quadratic equation in the object distance is obtained:

$$u^2 - uL + fL = 0 \tag{3}$$

with discriminant

$$D = L^2 - 4fL. \tag{4}$$

This means that there are two positions of the optical system, an enlarged image and a reduced image. Introducing a paramter $S = u_1 - u_2$, the difference between the solutions u_1 and u_2, gives

$$S = \sqrt{D}. \tag{5}$$

Rearranging (5) for the focal length yields Bessel's formula:

$$f = \frac{L^2 - S^2}{4L}. \tag{6}$$

2 Experimental Method

Figure 2: The apparatus consists of an optical rail with supports, an incandescent electric lamp, a translucent screen, and various converging and diverging lenses.

The aim this experiment was to determine the focal length of converging and diverging lenses using several different methods, each with varying degrees of accuracy. In the first part of the experiment, by placing a lens between an object and a screen, the position of the lens can be varied until a focused image is obtained.The object distance and image distance can be measured in each case, and by varying the position of the screen, a series of values is obtained. A graph of $\frac{1}{u}$ against $\frac{1}{v}$ can be used to determine the focal length of the lens.

The second uses Bessel's equation to find the focal length. First an enlarged image is obtained on the screen by moving the lens close to the object and adjusting until sharply focused. Then the lens is moved towards the screen and adjusted until a reduced image is focused on the screen. The difference in the position of the lens between the two focused images, S, is measured as is the total distance between the object and the image, L. Then the focal length can be found using Bessel's formula.

The third method involves determining the focal length using autocollimating. The converging lens is placed in front of the light source at a distance equal to the focal length. A transparent sheet with a grid pattern and a piece of white paper are each placed in front of half of the light source, and a plane mirror is placed behind the lens. Since the light source is at the focal point of the lens, the light rays will be reflected parallel to the optic axis, and will thus be reflected back at the same angle by the plane mirror. This causes an image of the grid pattern to form on the piece of paper. By adjusting the mirror until the image of the grid is the same size as the original grid, and adjusting the lens until as sharp an image as possible is obtained, the focal length is equal to the distance between the light source and the lens.

The fourth part of the experiment measures the focal length of a diverging lens by using it in combination with a converging lens. The position of the screen where the image is focused, S_1, is marked. Then a diverging lens is placed between the converging lens and the screen, and the position of the screen is adjusted until the image is once again focused, and this position, S_2 is measured. From the point of view of the diverging lens, the point S_1 is an object and the point S_2 is the image. The distance between S_1 and the diverging lens is noted as a_1 and the distance between S_2 and the diverging lens is noted as a_2. These are the object and image distances respectively, so rearranging (1), the focal length of the diverging lens can be determined using

$$f = \frac{a_1 a_2}{a_1 - a_2}. \tag{7}$$

Finally, the focal length was determined visually. A piece of paper was attached to the screen, and was rotated by 90° so the light rays could be seen to converge to a point (the focal point). The position is marked and the focal length of the converging lens is then obtained. This is repeated for a diverging lens, but this time the paper is folded in half, and the light rays are traced. the paper is removed from the screen and the traces are continued backwards until they converge to a point, determining the focal length of the diverging lens.

3 Results and Analysis

A table of values for the object and image distances was obtained.

	u(cm) ± 0.1cm	v(cm) ± 0.1cm
1	21.1	57.3
2	21.7	53.4
3	22.4	48.8
4	23.4	44.8
5	24.4	41.0
6	25.7	36.5

Table 1: Various measurments of object and image distances.

From the graph, the $y-$intercept obtained was $\frac{1}{f} = 0.071 \pm 0.002\text{cm}^{-1}$, giving a focal length of $f = 14.3 \pm 0.4$cm.

Using Bessel's equation, focal lengths of 9.64 ± 0.06cm and 14.92 ± 0.06cm were obtained for the 100mm and the 150mm lenses, respectively.
Using the autocollimating method, the focal lengths of various lenses were obtained. A table of the results is shown below.

By using the converging lens in combination with the diverging lens, the focal length of the diverging lens was found to be -10.7 ± 0.1cm.

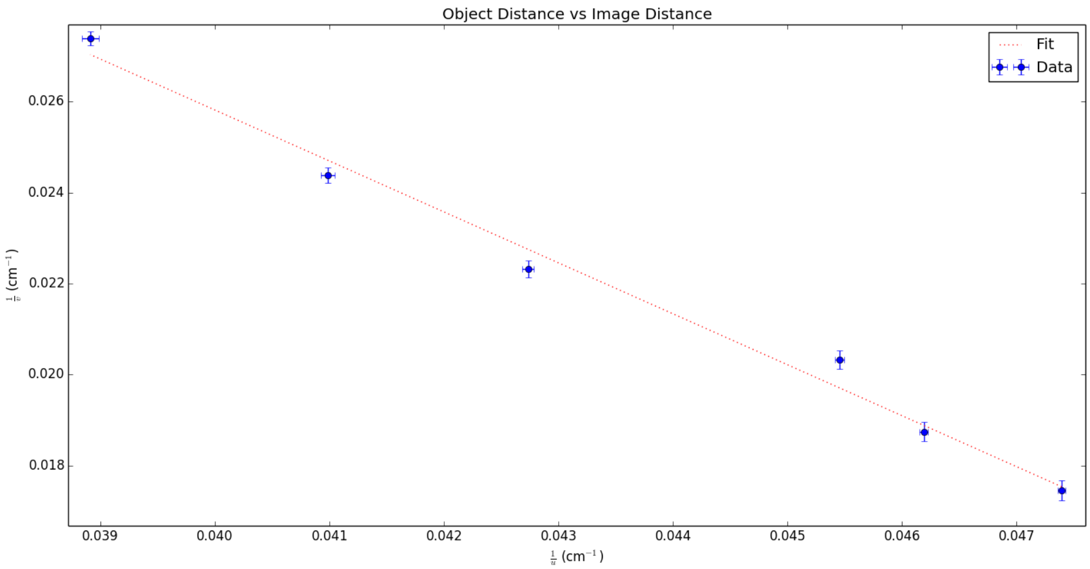

Figure 3: Inverse image distance vs inverse object distance. The intercepts give the inverse focal length

Lens (mm)	Focal Length (cm) ± 0.2cm
100	10.0
150	15.2
200	19.9
300	28.6

Using the piece of paper, the focal length of the converging lens was found to be 10.4 ± 0.1cm. The focal length of the diverging lens was found to be -8.6 ± 0.1cm.

4 Discussion and Conclusions

Our measured focal length of the lens for the first part from the graph was $f = 14.3 \pm 0.4$cm, and the actual focal length wasn't within the uncertainties of our value, possibly due to a small number of data points and errors in measurement. For all parts of this experiment, the allignment of the lens, light source and the screen may have been slightly offset, and the difference between the position of each device and the position of the edge of the support must also be taken into account. Values of 9.64 ± 0.06cm and 14.92 ± 0.06cm were found using Bessel's equation, which are quite accurate. The focal lengths measured using the autocollimating method were accurate to within the uncertainties except for the 300mm lens, therefore making it the most accurate method. Finding the focal lengths using a piece of paper was the least accurate method, especially in the case of the diverging lens.

References

[1] Eugene Hecht. Optics 4th edition. Optics, 4th Edition, Addison Wesley Longman Inc, 1998, 1, 1998.