Fraunhofer Diffraction at Diffraction Gratings

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September 14, 2015

1 Introduction and Theory

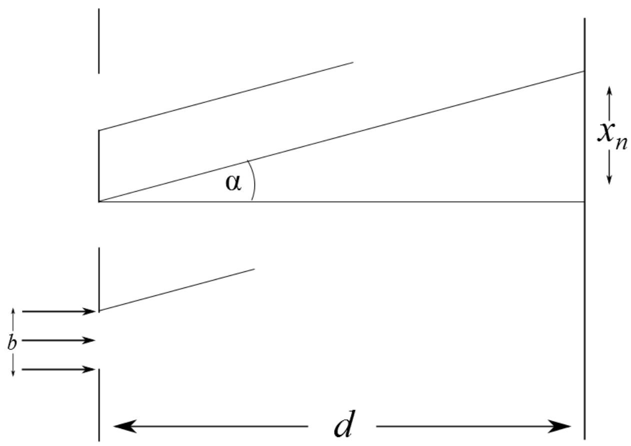

Figure 1: Diffraction in a grating.

Diffraction is a phenomenon that is observed when light waves are obstructed by an obstacle such as a slit. The behaviour of light under these circumstances is markedly different from the behaviour of light in geometric optics. The passing of light through a small slit causes the waves to spread outwards from the slit, deviating from its initial linear path. In classical physics, this is explained by the Huygens-Fresnel principle stating that each point on a wavefront is a point source of secondary waves and these sources interfere with one another.

There are two types of diffraction. Fraunhofer (far-field) diffraction occurs when the distance between the source and the diffraction object, as well as the distance between the diffraction object and the screen is extremely large, effectively infinite when compared with the wavelength of the incident light. In this case, light from a source of infinite distance away can be considered to be parallel plane waves when they are incident on the diffraction object. Fraunhofer diffraction is an approximation of Fresnel diffraction (near-field diffraction). Fresnel diffraction occurs when there is a finite distance between the source and the diffraction object, and once again from the diffraction object to the screen. In this case the waves cannot be considered to be parallel plane waves and the mathematics is considerably more complex. In this experiment, the distances are such that the Fraunhofer approximation for diffraction is valid.

In this experiment, the light is being diffracted by either a 1-dimensional or 2-dimensional grating. Assuming a grating with slit width b, period (distance between equivalent points of adjacent slits) d, and a distance between the grating and the screen of L, one can show that a maximum in the intensity of the interference pattern occurs at a point x_n and is related to the wavelength of the light λ, d and L as follows:

$$x_n = n\frac{\lambda L}{d}, \tag{1}$$

where n is an integer. This is an approximation due to the fact that L is much larger than d and therefore $\sin(\theta) \approx \tan(\theta) \approx \theta$. By measuring the distance between the centre of the pattern and the first maximum, the slit spacing of an unknown grating can be determined.

The intensity distribution of a diffraction pattern produced by a grating of N slits of width b and spacing d is a product of two terms; the first term describing a sequence of maxima and minima that would be observed with a grating of N slits with infinitessimal width, and the second term describing the diffraction pattern observed from one slit of finite width. The second term has a much slower and broader single peak whereas the first term has multiple maxima and minima. The second term can be thought of as the envelope function that the first term must oscillate inside. The expression is as follows:

$$I(\alpha) = \frac{\sin^2\left(\frac{\phi}{2}\right)}{\left(\frac{\phi}{2}\right)} \frac{\sin^2\left(\frac{N}{2}\varphi\right)}{\sin^2\left(\frac{\varphi}{2}\right)} \tag{2}$$

where $\phi = \frac{2\pi}{\lambda} b \sin(\alpha)$ and $\varphi = \frac{2\pi}{\lambda} d \sin(\alpha)$.The mathematics for a 2-dimensional grating, which is simply 2 1-dimensional gratings overlapped at 90°, is almost identical and can be described by the same equation, this time the equation is used with the superposition of 2 perpendicular patterns.

2 Experimental Method

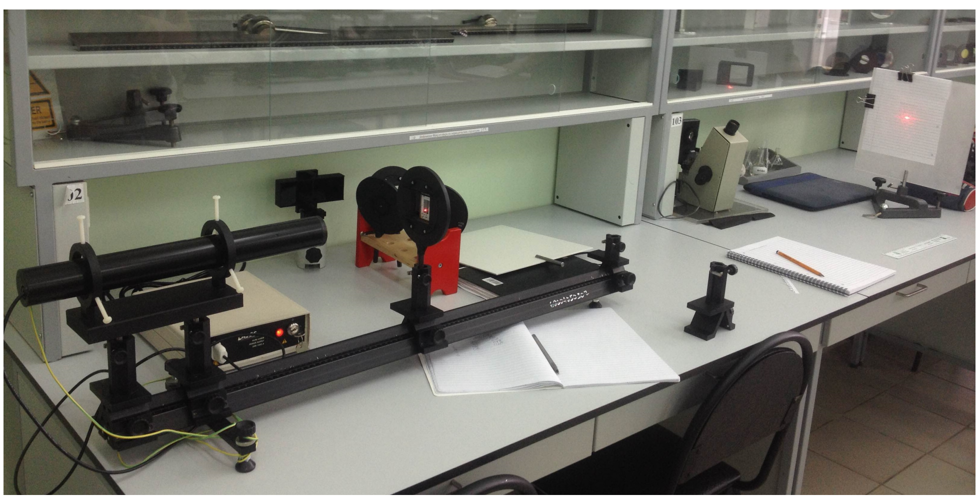

Figure 2: Experimental setup.

The apparatus was set up as shown in the diagram; the HeNe laser was mounted on one end of the optical bench and a holder with spring clips was placed roughly 20cm in front of the laser. A sled tripod was used to set up a screen so that it was positioned roughly 170cm in front of the holder with spring clips. The laser was switched on and the beam directed onto the centre of the screen. In part one of this experiment the diaphragm with the three 1-dimensional gratings, each of different slit spacing, was placed in the holder and secured using the spring clips. The plate was moved within the holder such that the laser beam fell solely on one of the three gratings and the resulting diffraction pattern was observed on the screen. The diffraction patterns resulting from the passage of the laser beam through the other two gratings were also observed by moving the plate within the holder. A sheet of paper was secured to the screen by means of a set of clips and after selecting one of the three gratings to begin with, the positions of maximum light intensity were marked on the paper using a soft pencil. The sheet of paper was removed from the screen and the distances between the maxima were measured using a ruler. This procedure was repeated for each of the three gratings of different slit spacing. Finally the distance L between the grating and the screen was measured using a measuring tape. This L was held constant during the course of all measurements taken.

In the second part of the experiment, the diaphragm with the 1-dimensional grating was removed from the holder and replaced with the diaphragm containing two 2-dimensional gratings. By shifting the position of the plate within the holder, the diffraction pattern resulting from the passage of the laser beam through each of the gratings were observed on the screen. Again, beginning with one of the gratings, a sheet of paper was secured to the screen and the positions of the intensity maxima were marked using a pencil. The paper was removed and the distances between the maxima were measured. This process was repeated for the second grating. The distance L between the diffraction grating and the screen was also measured using a tape measure. Both gratings had a slit spacing 0.25mm.

3 Results and Analysis

The measured values of the slit widths are recorded in the table below.

Slit spacing	x (mm)	d (m)
0.5	8	5.43×10^{-4}
0.25	4	2.72×10^{-4}
0.125	2	1.36×10^{-4}73.3

For the second part of the experiment, a value of $\lambda = 582 \pm 146$nm was obtained for the wavelength of the HeNe laser.

4 Discussion and Conclusions

In the first part of the experiment, we found the width of the slits of the 1-dimensional gratings to be $1.36 \times 10^{-4} \pm 1.7 \times 10^{-4}$m, $2.72 \times 10^{-4} \pm 6.8 \times 10^{-5}$m and $5.43 \times 10^{-4} \pm 2.8 \times 10^{-4}$m. These compare very well with the values quoted on the gratings themselves of 1.25×10^{-4}m, 2.5×10^{-4}m and 5×10^{-4}m. All quoted values were within our error margins and our measured values were all quite close to the quoted values. With these results, it is clear that the apparatus is functioning correctly and that our experimentally recorded values agree well with the theory. The pattern in this case had a large maximum of brightness at the centre surrounded on either side by smaller regularly spaced maxima and minima.

In the second part of the experiment, it was assumed that the quoted value for the spacing between slits on the grating was 0.25mm. Under this assumption we calculated the wavelength of the laser light used in the experiment to be $\lambda = 582 \pm 146$nm. The quoted value for the wavelength of the HeNe laser used was 632.8nm. Our results agree with this figure as the figure is well within our margin of error. Qualitatively, the pattern for this part of the experiment looked very similar to the pattern produced by the one dimensional slit, except for the fact that there were effectively two diffraction patterns; one extending horizontally and one extending vertically.

There were several sources of inaccuracy that could easily be improved in this experiment in order to reduce our errors and to improve our recorded values.

Firstly, the laser was emitting a point beam, but there was also a considerable amount of noise or excess light that was being emitted from the laser. This caused the diffraction pattern to become considerably more difficult to see on the screen as there were a large amount of extra spots that were obscuring the pattern from view. In several cases, this made it difficult to determine exactly where some of the maxima or minima were as there was so much excess light. This was particularly apparent in the case where the spaces between the maxima was small. This could be resolved by simply using a more collimated light source where the beam was more focused and less excess noise was emitted.

Secondly, there were several large scratches and cracks in the diffraction grating. In general, it was possible to pass the laser through a point on the diffraction grating where there were no cracks, however, these cracks may well have caused a slight offset in the spacing between the slits and these could very easily have affected our results. The solution to this problem is simply to repeat the experiment with undamaged filters.

The third source of inaccuracy was that the optical rail was not long enough to mount the screen a suitable distance from the gratings. The experiment calls for 170cm between the slits and the screen. The optical rail is only 100cm long. This meant that the screen had to be mounted on a tripod that was not connected to the optical rail. This meant that the alignment of the screen was difficult, the measurement of the distance between the grating and the screen was more inaccurate as the measuring tape may not have been exactly straight or exactly horizontal. Also, the fact that the tripod was not connected to the optical rail meant that when attempting to measure the distance between maxima, it was very easy to move the screen from its initial position, thus invalidating the measurement taken for the distance between the slit and the screen. These errors would have propagated through our calculations and produced inconsistent results.

In conclusion, despite the sources of error mentioned above, all of our recorded measurements match the quoted values within our margin of error and all of the aims of this experiment were completed successfully.