

The number of microstates is:

$$\Omega = \frac{N_L!}{N_V!(N_L - N_V)!}$$

where  $N_V$  is the number of defects, and  $N_L$  is the number of atoms.

Entropy is:

$$S = k_B \ln \Omega$$

Using the approximation  $\ln x! \approx x \ln x$ , this gives:

$$\Delta S_m = k_B [N_L \ln N_L - N_V \ln N_V - (N_L - N_V) \ln(N_L - N_V)]$$

The change in free energy is:

$$\Delta F = N_V \Delta h_V + T \Delta S$$

where  $\Delta h_V$  is the energy cost of creating one vacancy.

So

$$\Delta h_V + k_B T \ln \left( \frac{N_V}{N_L - N_V} \right) = 0,$$

or, since  $N_V \ll N_L$ ,

$$\begin{aligned} \Delta h_V + k_B T \ln \frac{N_V}{N_L} &= 0, \\ \Rightarrow \frac{N_V}{N_L} &\approx \exp \left( -\frac{\Delta h_V}{k_B T} \right). \end{aligned}$$