Brian Conway 08354502, Nov '10

The Hall Effect

Abstract:

The Hall effect was examined for a germanium crystal placed in a electromagnet. The hall coefficient was determined to be $-0.0122(\pm 0.0009)m^3C^{-1}$. The charge carriers were determined to be electrons. Also determined were the carrier concentration $N=5.16(\pm 0.04)E\,20m^{-3}$ **,** electrical conductivity $\sigma=1.82(\pm 0.03)(\Omega m)^{-1}$, carrier mobility $\mu=\sigma/Nq=\sigma R_H$.
The hall effect was also examined using a permanent magnet in a 'magic cylinder'. Figures broadly in line with those found using the first method were obtained.

Introduction:

The hall effect was discovered by Edwin Hall in 1879, 18 years before the discovery of the electron.

-The effect occurs due to how current flows in a conductor. Current is the movement of electrons or holes.
-When a magnetic field is applied perpendicular to the current, it produces a Lorentz force on the charge carriers, which causes the carriers to deflect from their straight-through direction (from 1 to 2, see figure 1).
-This will result in a build up of carriers on side 3, say. This build up of charge will be balanced by an equal and opposite charge on side 4.
-This produces an electric field (Hall field $\vec{E}_H$) between 3 and 4, that will balance out with the Lorentz force ($\vec{F}$), which acts in opposing direction.
-When these two forces are balanced, the current will again travel from 1 to 2 unimpeded.

The Lorentz force is given by;

$\vec{F}=q\vec{V}\,x\,\vec{B}$, where $\vec{V}$ is the drift velocity of current between 1 and 2, and q is charge of each carrier

The Hall field is given by;

$\vec{E}_h=R_H\vec{B}\,x\,\vec{J}$,where R_h is the Hall coeffecient, and $\vec{J}$ is current density between 1 and 2.

The hall voltage is $V_H=E_H w$ and the current $\vec{I}=wt\,\vec{J}$, therefore we can see $V_H=R_H BI/t$

$R_H=1/Nq$, where N is the number of carriers per unit volume, each of charge q.

It is important to note, that the type of charge carrier gives a different sign of $\vec{E}_H$. Therefore, the sign of $\vec{R}_H$ determines what type of carrier is present.

The electrical conductivity(σ) for one type of carrier is given by;

$\sigma=1/\rho$, where ρ is the resistivity

$\sigma=Nq\,\mu$, where μ is the carrier mobility, which is a measure of how easily a carrier moves through the material.

Part 1:

-Experimental Details

The characteristics of an electromagnet are investigated in this experiment
An electromagnet has a core made from a ferromagnetic(soft) material. When the current in coil $\vec{I}_C$ around core is reduced to zero, the magnetism in core is not quite reduced to zero, but leaves a small remnant field, B_r .

The magnetic field, $\vec{B}$ is measured in the gap between the two coils, see figure 2.
A hall probe gaussmeter is inserted between the gap to measure the field.
-Rotate the probe into the correct orientation that gives the maximum reading of field.
-Measure B and I_C for I_C varying from 0V→2.5V→0V→-2.5V→0V
-Plot B versus I_C

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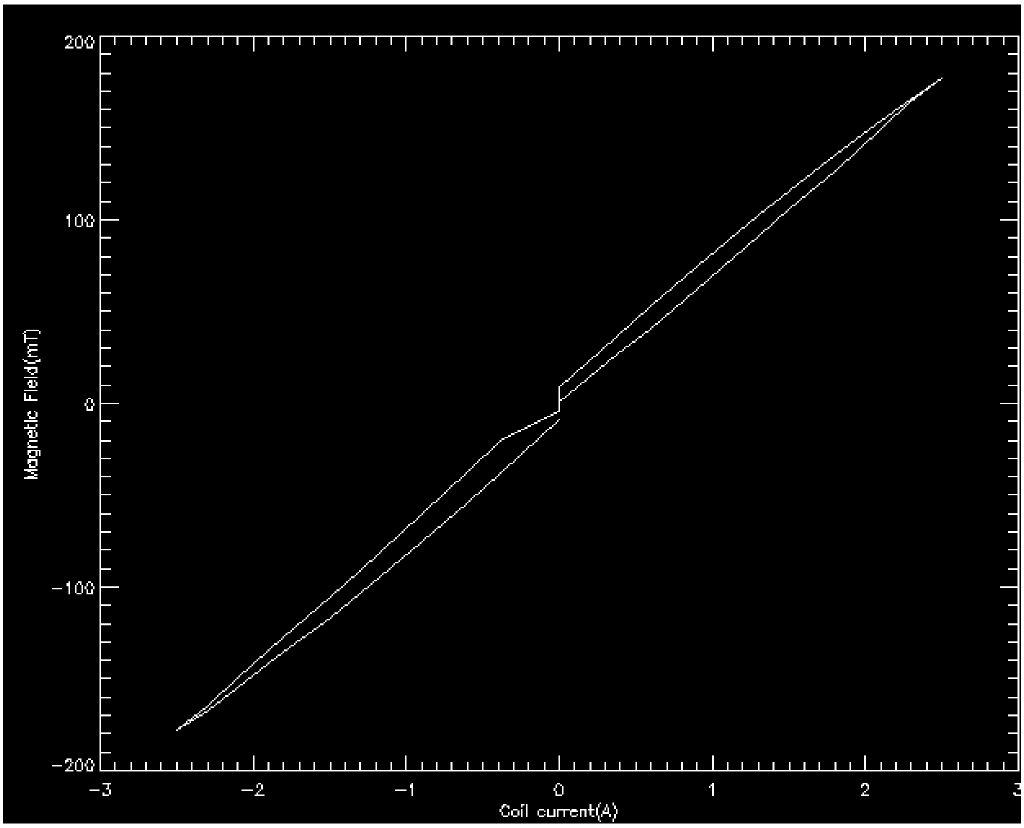
-Results and Analysis



Plot 1

From the plot it is easy to see that there is no unique value B for a given I_C and therefore it is not possible to provide a unique calibration of B vs I_C .
The remnant field in this case is 16mT, but this value was dependent on maximum value for I_C that was obtained.
When we began the experiment the current was zero, as was the magnetic field, since the electromagnet had been left uncharged for a long time.

The current was increased from 0 to 2.5A. When the current was decreased back to 0A, different values for B were obtained for the same values of I_C , since the remnant field in the electromagnet, from being charged to 2.5A, added each value of B.

This resulted in a loop as seen in *plot 1.* This loop is unique to the maximum value that I_C is charged to.

Conclusion

It was found that a electromagnet is left with a remnant field after been charged with a current I. $\vec{B}$ was found to be proportional to I, but this relation could not be relied on to take measurements and it was concluded that the field $\vec{B}$ needs to be measured directly for every measurement taken.

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Part 2:
-Experimental Details
"Measurements of V_H using the electromagnet"

The alignment of terminals 3 and 4 is not perfectly orthogonal to alignment of 1 and 2, so this must first be corrected for
-the misalignment of 3 and 4 means that some additional voltage V_0 is added to V_{34} ;

$$V_{34} = V_H + V_0$$

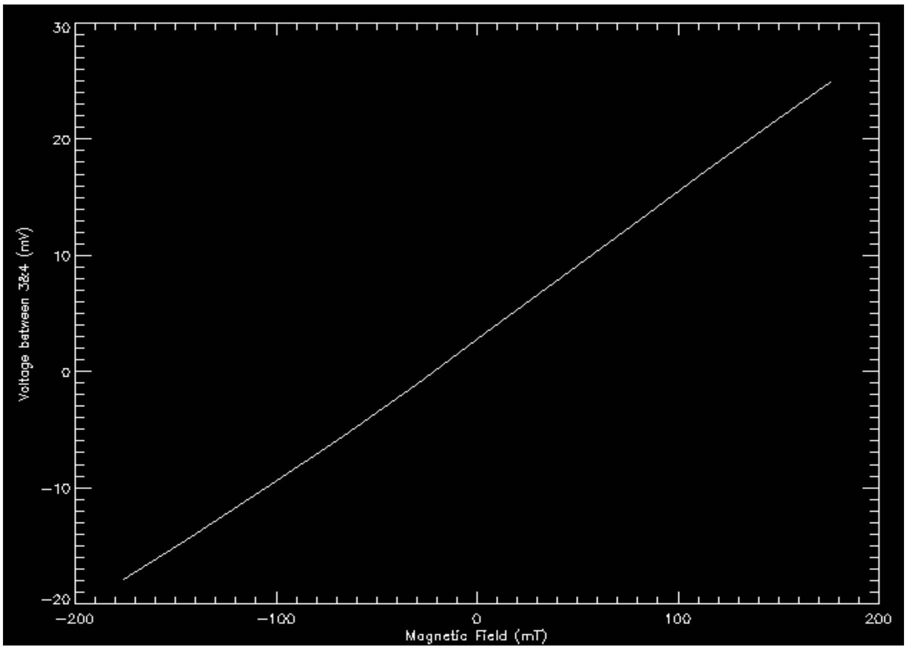
-this can be corrected for by adjusting the potentiometer on the circuit board, while B=0T.
-set I=10mA, B=0T and adjust potentiometer until V_{34} (as measured with a DMM) is zero.

The following diagram is the set-up for part 2.

-Results and Analysis

For a given I value, V_0 will be constant. Therefore to show $V_H \alpha B$, it need only be shown that $V_{34} \alpha B$, by measuring V_{34} and plotting it against B, for a given I.

The direction of $\vec{B}$ was found using a compass. The result of this implies that the values of $\vec{B}$ obtained should have been opposite charge of following data. This has been accounted for in the value's given for R_H and any subsequent result obtained using these value's.

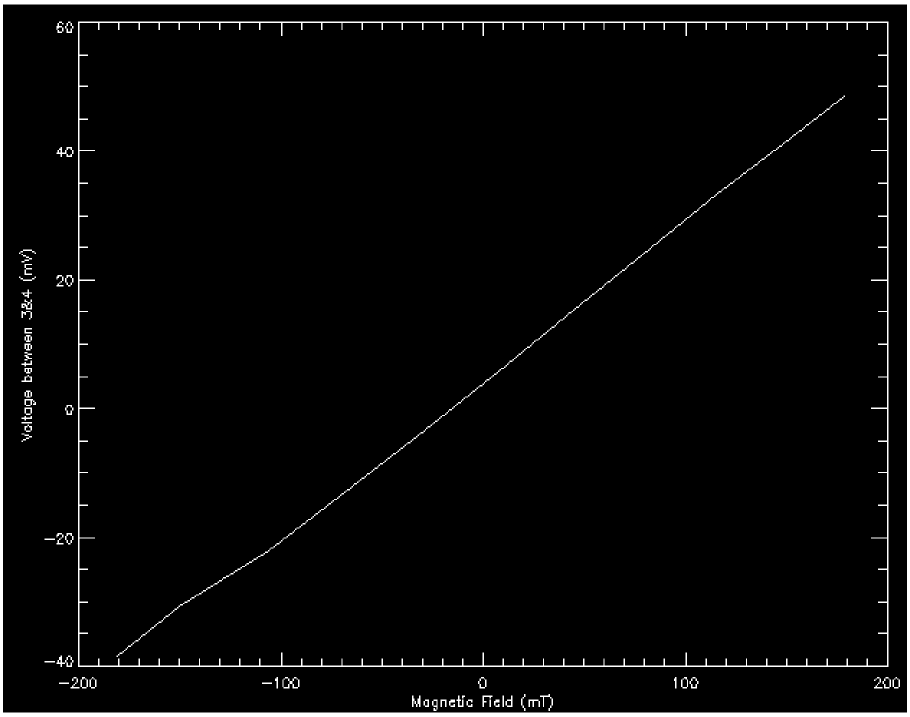


V_{34} against B for I=10.14mA

The slope of this graph is found to be 0.12(±0.03)V/T by using an idl program. The slope of the graph is;

$$m = R_H I / t \quad .$$

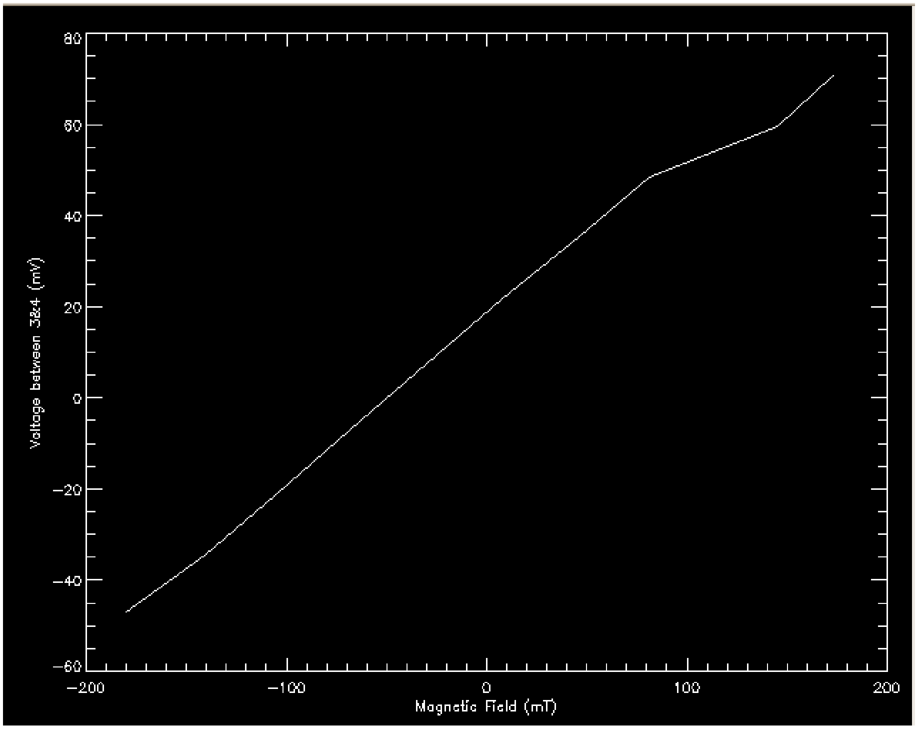
Therefore, R_h is calculated to be $-0.012(\pm 0.001) m^3 C^{-1}$



V_{34} against B for I=20.4mA

The slope of this graph is found to be 0.24(±0.03)V/T.

R_h is calculated to be $-0.011(\pm 0.001) m^3 C^{-1}$



V_{34} against B for I=30.

The slope of this graph is found to be 0.37(±0.03)V/T.

R_h is calculated to be $-0.0122(\pm 0.0009) m^3 C^{-1}$

It is obvious from the 3plots above of V_{34} against B that $V_H \alpha B$.

The average value of R_H is found to be $-0.0121(\pm 0.0001) m^3 C^{-1}$.

The carrier concentration is now easily obtained to be;

$$N = 5.16(\pm 0.04) E 20 m^{-3}$$

The sign of R_H was obtained to be negative from the plotted data. From the equation;

$R_H = 1/Nq$, it can be deduced that q is negative and ***electrons*** are the charge carriers.

The conductivity, $\sigma = 1/\rho$, can be determined by measuring the resistance across 1 and 2, and hence determining ρ from $\rho = R(wt)/l$.

$\rho = 0.55(\pm 0.01)\Omega m$,

$\sigma = 1.82(\pm 0.03)(\Omega m)^{-1}$,

Finally, the charge mobility can be determined;
$\mu = \sigma / Nq = \sigma R_H$

$\mu = 0.0220(\pm 0.0004) m^2 (Vs)^{-1}$

Conclusions

It was found that the charge carriers for a germanium crystals are electrons. The hall coeffecient was found to be $-0.0121(\pm 0.0001) m^3 C^{-1}$. The carrier concentration was found to be $N = 5.16(\pm 0.04) E20 m^{-3}$, electrical conductivity was found to be $\sigma = 1.82(\pm 0.03)(\Omega m)^{-1}$ and the carrier mobility $\mu = 0.0220(\pm 0.0004) m^2 (Vs)^{-1}$.

Part 3:
-Experimental Details

Part 3 examines the hall effect for a germanium crystal in a rotating permanent magnet, 'magic cylinder'.
Figures can be obtained by measuring directly using DMM's and also using an oscilloscope.

Part 3 also examines the characteristics of a lock-in amplifier.

The lock-in amplifier changes an A.C. input to a D.C. output by taking the root mean squares value of the A.C. It also filters out noise by removing all frequencies of a signal that are a certain small band away from the frequency centre.

It can be seen below how the lock-in amplifier works. At a certain point along the signal the polarity is changed and the output is reversed. This is done at the every wavelength at the same point on the wave.

Figure 1 shows how a maximum D.C. output is obtained, while firgure 2 shows how a minimum output is obtained. The phase difference between min and max is 90 degrees.

Results and Analysis

Figures obtained by measuring data directly using DMM's;

$I(mA)$	$V_H(mV)$	$V_0(mV)$	V_0/V_{12}	$R(\Omega)$
5.00 (±0.01)	8.9	11.2	0.021	109.2
10.00(±0.01)	19.5	23.5	0.021	111.1
15.00(±0.01)	30.7	36.4	0.022	111.4
20.0(±0.1)	41	51.5	0.023	112.5
25.0(±0.1)	51.2	67.3	0.024	112.8

The sign of R_H is found by examining the magic cylinder. When $\vec{B}$ is perpendicular to the sample V_{34} is a maximum. It is deduced from the maximum position that V_{34} goes from 4 to 3, therefore electrons are the charge carriers.

R_H was found to be $-0.0104(\pm 0.0005)m^3C^{-1}$, again using an idl program.

N was found to be $6.00E20(\pm 0.06)m^{-3}$.

$\sigma = 17.9(\pm 0.3)\Omega m$

$\mu = 0.186(\pm 0.001)m^2(Vs)^{-1}$

Figures obtained using an oscilloscope;

$I(mA)$	$V_{34}(mV)$	$V_{12}(mV)$
5.00 (±0.01)	20.0(±4.8)	0.5629(±0.001)
10.00(±0.01)	40.0(±5)	1.126(±0.001)
15.00(±0.01)	61.6(±5)	1.78(±0.01)
20.0(±0.1)	81.6(±8)	2.28(±0.01)
25.0(±0.1)	102.0(±8)	2.84(±0.01)

$R_H = -0.0118(\pm 0.0007)m^3C^{-1}$

$N = 5.29E20(\pm 0.07)m^{-3}$

$\sigma = 17.6(\pm 0.3)\Omega m$

$\mu = 0.208(\pm 0.002)m^2(Vs)^{-1}$

Conclusions

The charge carriers for a germanium crystal in a 'magic cylinder' were found to be electrons, as before. R_H was found to be $-0.0104(\pm 0.0005)m^3C^{-1}$ and $-0.0118(\pm 0.0007)m^3C^{-1}$ by taking measurements using a DMM and an oscilloscope respectively. The figures for electrical conductivity, $17.9(\pm 0.3)\Omega m$ and $17.6(\pm 0.3)\Omega m$,and carrier mobility $0.186(\pm 0.001)m^2(Vs)^{-1}$ and $0.208(\pm 0.002)m^2(Vs)^{-1}$ were about a factor of 10 bigger than in those obtained in part 2. This is due to how the rotating magnetic field affects the carriers movement in the crystal.

The lock-in amplifier was found to be a very useful device that can have several applications.

Appendix

idl syntax for part 2 for current equal to 10mA;

```
“pro hall_2_I_10kai

B=[176, 146, 114, 81, 46, 8, -30, -67, -105, -142, -176]
B=float(B)

V=[24.9, 21.3, 17.3, 13.1, 8.6, 3.8, -1.1, -5.6,  -10, -14.2, -17.9]
V=float(V)
```

```
measure_errors = REPLICATE(10.0, N_ELEMENTS(V))

result = REGRESS(B, V, SIGMA=sigma, CONST=const, MEASURE_ERRORS=measure_errors)

I=10.14E-3
dI=0.01E-3
t=1E-3
dt=0.02E-3
Rh=(result*t)/I

dRh=Rh*(sqrt(   (dt/t)^2 + (sigma/result)^2 + (dI/I)^2))

print, 'Constant:', const
print, 'Slope:', result
print, 'Δslope', sigma
print, 'Rh', Rh
print, 'ΔRh', dRh
end”
```

idl syntax for part 3 by using DMM's;

```
“pro hall_3_1

B=[170]
dB=10
t=1E-3
dt=0.02E-3

I=[5.00, 10.00, 15.00, 20.0, 25.0]
dI=[0.01, 0.01, 0.01, 0.1, 0.1]

I=float(I)

V34_Bp=[20, 43, 67, 92.4, 118.4]
dV34_Bp=0.3
V34_Bp=float(V34_Bp)

V34_Bm=[2.3, 4, 5.7, 10.5, 16.1]
dV34_Bm=0.3
V34_Bm=float(V34_Bm)

V12=[0.546, 1.111, 1.671, 2.25, 2.82]
V12=float(V12)

Vh=(0.5)*(  V34_Bp - V34_Bm  )

dVh=sqrt(  (dV34_Bp)^2 +  (dV34_Bm)^2 )

V0=(0.5)*(  V34_Bp + V34_Bm  )

V34= Vh + V0

measure_errors = REPLICATE(10.0, N_ELEMENTS(V34))

result = REGRESS(I, V34, SIGMA=sigma, CONST=const, MEASURE_ERRORS=measure_errors)

Rh=Vh*t/(B*I)

dRh=Rh*(sqrt(   (dVh/Vh)^2 + (dt/t)^2 + (dB/B)^2) + (dI/I)^2  )

print, 'Constant:', const
print, 'Slope:', result
print, 'Δslope', sigma
print, 'Rh', Rh*1000
print, 'ΔRh', dRh*1000

plot, I, V34, xtitle='current (mA)', ytitle='Voltage between 3 &4 (mV)'

print, 'Vh=', Vh
print, 'V0=', V0
print, 'V34=', V34
print, 'V0/V12=', V0/V12

end”
```