Models for wealth distribution in societies

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ABSTRACT

Income distributions are of great interest to governments and people at large. Recently physicists have realized that they too can begin to understand their nature and dynamics using established methods. This poster illustrates how modern physics is being used to model such distributions. The models all assume that people or agents can exchange money by analogy to momentum or energy exchange processes between molecules. In the Lotka-Volterra model [1] agents are placed on a fixed lattice and money is exchanged according to an algorithm. This can be generalized to a family-network model [4], where the structure of the network of economic interactions is not predefined, but emerges from the wealth dynamics. Another class of models treats the agents by analogy to molecules in a gas [2, 3]. Here the agents can save and exchange money. The stationary distribution of wealth for all these models shows characteristics that mimic real income distributions. In particular, the models all predict the well known power-law tail for large values of wealth, first discovered by Pareto, 100 years ago.

1 Lotka Volterra Model

The generalized Lotka-Volterra system describes the evolution in discrete time of the N dynamic variables w_i, $i = 1, ..., N$. In economic systems, w_i represents the wealth of an individual in a society.

Three main mechanisms are included in these systems:

- A stochastic autocatalytic term representing investments
- A drift term representing social security payments
- A time dependent saturation term due to the finite size of economy

At each time step t, an agent is chosen at randomly and his wealth is updated:

$$w_i(t+1) = [1+\lambda(t)]w_i(t) + \sum_{j=1}^{N} a_{ij} w_j(t) - \sum_{j=1}^{N} a_{ji} w_i(t) - \sum_{j=1}^{N} c_{ij} w_i(t) w_j(t)$$

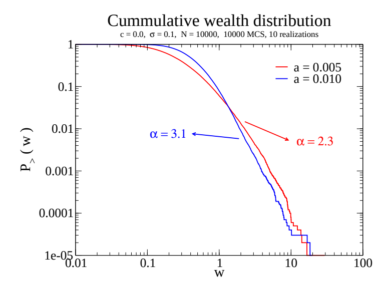
Analytic results give a Pareto tail: $\alpha = 1 + \frac{2a}{\sigma}$.

We have extended this to a model where interaction takes a place on a 2-dimensional lattice, taking into account that in a real situation exchanges are not fully random, but happen within some sort of network:

- N agents in a 2-dimensional lattice with size L x L
- Each agent trades with his 4 next neighbors
- $a_{ij} = a$ and $c_{ij} = c$ for all i and j

$$w_i(t+1) = [1+\lambda(t) - 4a]w_i(t) + [a - c w_i(t)] \sum_{j \in N_i} w_j(t)$$

1.1 Wealth Distribution for 2D lattice



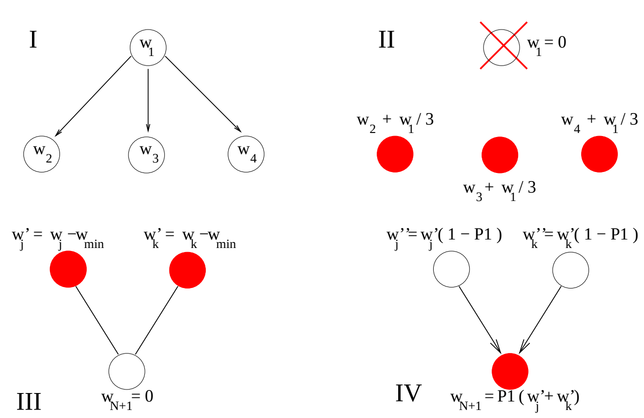
Power law tail that changes with a. The value of the power law is different from the exponents in the generalized Lotka Volterra model. Analytical studies for the 2-dimensional model are needed to compare with the numerical results.

2 Family Network Model

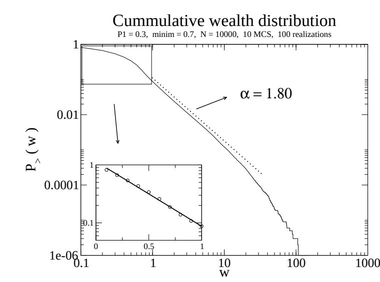
The Family Network Model is a conservative model of wealth exchange where the main mechanisms of wealth transfer are "inheritance" and "social costs" associated with raising a new family. The structure of the network of social (economic) interactions is not predefined but emerges from the wealth dynamics.

2.1 Algorithm

- The "oldest" node dies and distributes his wealth by his family
- A new node born and it is attached to two nodes in the system that have wealth over the W_{min} value
- The W_{min} value is subtracted from each of the two families and is divided in equal amounts and preferentially redistributed in the system
- The two parent families will give a $P1$ portion of their wealth to help the start-up of the new family
- Everything is repeated for all the agents for many Monte Carlo steps



2.2 Wealth Distribution



Power law in agreement with empirical data which generally has exponents between 1 and 2.

3 Gas Model

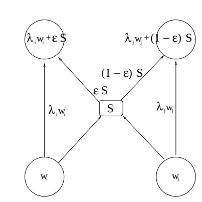
In the gas model, agents act like molecules and are chosen at random to interact with each other. Every time that we choose a pair of agents, a trade happens corresponding to the exchange of energy in the collisions of atoms [2]. In this model, the agents are allowed to save various proportions of their money. This leads to the formation of a power law tails, i.e. some agents become very rich.

3.1 Rules of trade

- N agents with wealth w_i, $i = 1, ..., N$, that are allowed to save a fraction λ_i of their money prior to an interaction
- Money is conserved during interactions

$$w_i(t+1) = \lambda_i w_i(t) + \epsilon[(1-\lambda_i)w_i(t) + (1-\lambda_j)w_j(t)]$$
$$w_j(t+1) = \lambda_j w_j(t) + (1-\epsilon)[(1-\lambda_i)w_i(t) + (1-\lambda_j)w_j(t)]$$

- ϵ is a random number between $[0, 1]$, for the random division in the interaction



3.2 Results

- No savings ($\lambda_i = 0$ for all i), the distribution follows the Gibbs rule:

$$P(w) \sim exp\left(-\frac{w}{\overline{w}}\right)$$

- For $\lambda_i \neq 0$ ($\lambda_i = \lambda$ for all i), the resulting distribution can be fitted by the function:

$$P(w) = \frac{n^n}{\Gamma(n)} w^{n-1} exp(-nw)$$

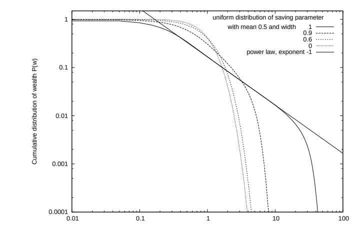
- $\Gamma(n)$ is a Gamma function and n is a parameter related to the saving propensity λ: $n(\lambda) = 1 + \frac{3\lambda}{1-\lambda}$

- For λ_i chosen from some random distribution, the distribution of money for large values is given by:

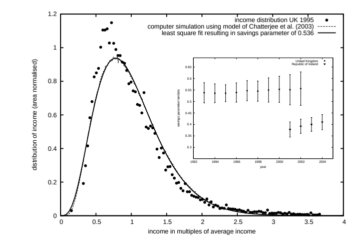
$$P(w) \sim w^{-1-\alpha}$$

- The exponent $\alpha = 1$ from analytical solutions and simulations

3.3 Wealth Distributions



Numerical simulations (500 agents and 39000 realizations, each taken after 100000 equilibration steps) agree with the analytical studies.



The model may be used to describe empirical data. The only free parameter is the average saving propensity, which is shown to be constant over time.

4 Conclusions

- A more detailed study of the role of the parameters for the 2-dimensional Lotka Volterra Model is needed
- Power Law in the richest part of wealth distribution of the Family Network Model is in agreement with empirical data
- Analytic study of the distributions of the Gas Model agrees with the numerical one
- These models give guidance for the implementation of social taxes. In addiction, a non-existing Pareto tail in the real wealth data might be an indicator of tax fraud.

References

[1] Solomon, S. and Richmond, P., Physica A **299**, 188 (2001).

[2] Chatterjee, A., Chakrabarti, B. K. and Manna, S. S., Physica A **335**, 155 (2004)

[3] Repetowicz, P., Hutzler, S. and Richmond, P., preprint cond-mat/0407770

[4] Coelho, R., Zoltan, N., Ramasco, J.J. and Santos, M.A., preprint cond-mat/0412516