Dynamics of correlations from a FTSE100 portfolio

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1. INTRODUCTION

Since the work of Mantegna [1], where the author introduces a metric distance related to the correlations between stocks, and represents the taxonomy of the market as a network of stocks (Minimal Spanning Tree, MST), much has been done in trying to understand how this network of stocks gives us informations about the market, and how the network changes with time [2, 3, 4].

2. Taxonomy of the Market

Comparing the time series of log-returns, we study the correlation coefficient between pairs of stocks. From these correlations we can compute a distance, for each pair, that enables us to construct a network with links between stocks.

2.1 Correlations

The correlation coefficient is a relation between the time series of different stocks:

$$\rho_{ij}(t) = \frac{< R_i R_j > - < R_i >< R_j >}{\sqrt{\left(< R_i^2 > - < R_i >^2\right)\left(< R_j^2 > - < R_j >^2\right)}}$$

where $R_i(t) = \ln P_i(t) - \ln P_i(t-1)$ is the log-return and $P_i(t)$ the daily closure price of stock i at time t.

2.2 Distances

The distance introduced by Mantegna, is determined from the Euclidean distance between vectors that are the time series of log-returns. It is related to the correlation coefficient:

$$d_{ij}(t) = \sqrt{2\left(1 - \rho_{ij}(t)\right)}$$

The matrix elements have values between $0 \leq d_{ij} \leq 2$. The distance matrix has the same properties as the correlation matrix.

2.3 Minimal Spanning Trees

The distance matrix is now used to construct the MST. The MST has $N-1$ edges, connecting N stocks, such that the sum of all distances (tree length) is minimum. We use the Prim's algorithm to compute the MST.
The portfolio investigated here is composed of 67 stocks from FTSE100 with time series length of about 9 years (2322 days) from August 1996 until June 2005. Figure 1 represents the MST for the 2322 days, using the new classification adopted by FTSE since the beginning of 2006, the Industry Classification Benchmark created by Dow Jones Indexes and FTSE, where each symbol represents a different industry: Oil & Gas (■), Basic Materials (△), Industrials (⧫), Consumer Goods (gray □), Health Care (□), Consumer Services (▲), Telecommunications (◊), Utilities (●), Financials (gray ○) and Technology (○).
The new classification adopted by FTSE in January 2006 clearly mimics much more closely the MST results, compared with the old classification [5]. The implementation of the new supersector groups ensures that apart from some notable exceptions stocks from the same supersector are now connected.

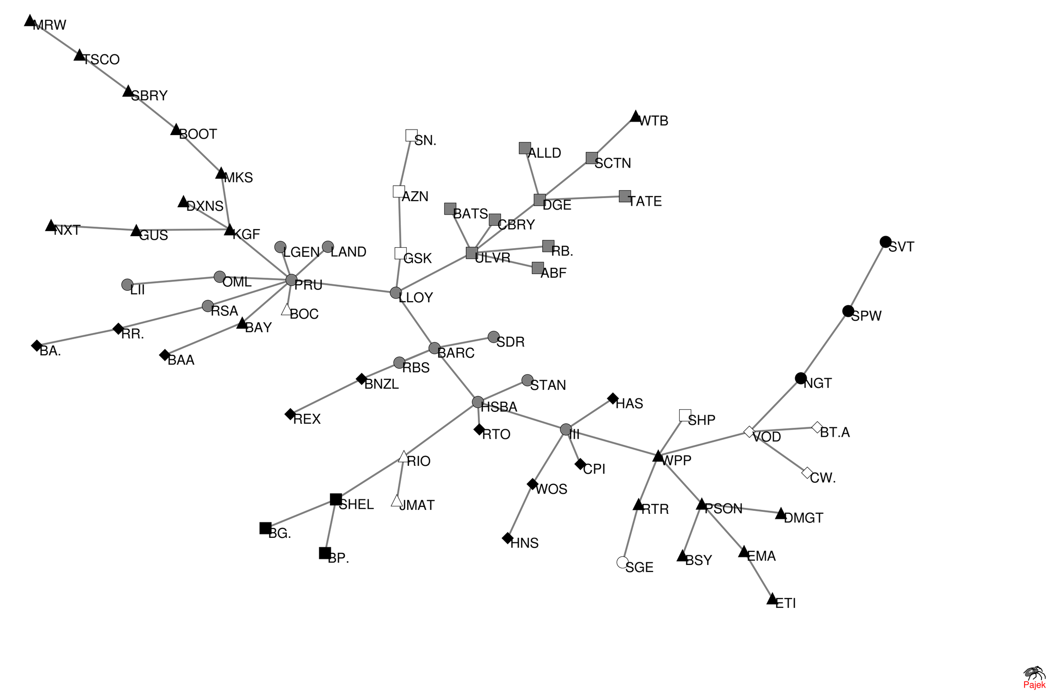

FIGURE 1: MST for time series of 2322 days and 67 stocks from the FTSE100 index.

3. Random Time Series

If we model the returns as random numbers from a specific distribution, we can compute the correlations, distances and trees for this random series. Trying to create random time series with more real characteristics than a completely Gaussian random time series, we introduce a control term (the return of FTSE Index) for the generation of random time series:

$$R_i(t) = \alpha_i + \beta_i R_m(t) + \epsilon_i(t)$$

where α_i and β_i are real parameters (estimated by the least square method), $R_m(t)$ is the market factor (return of FTSE Index) and $\epsilon_i(t)$ is the stochastic variable from a Gaussian distribution with variance σ_i. This model is known as the Market Model. The MST for random time series created with the Market Model is represented in Figure 2.

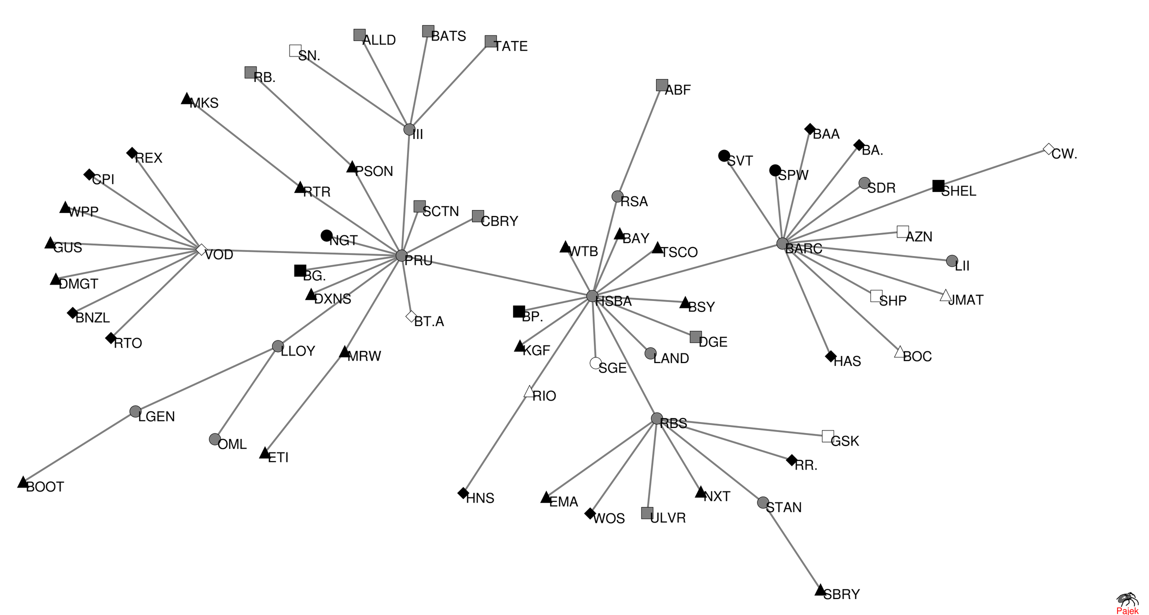

FIGURE 2: MST for random time series (from a Market Model) of 2322 days and 67 stocks.

In the Random MST, we see that the stocks from the Financial group (gray ○), are all linked together. As in the MST for real data (Figure 1) they act as the backbone of the network. However, the presence of 6 nodes with up to 13 links differs from the topology of real data.

4. Dependence in time

We can study the variation in time of the moments of both correlation matrix and distances that are part of the MST (Figure 3).

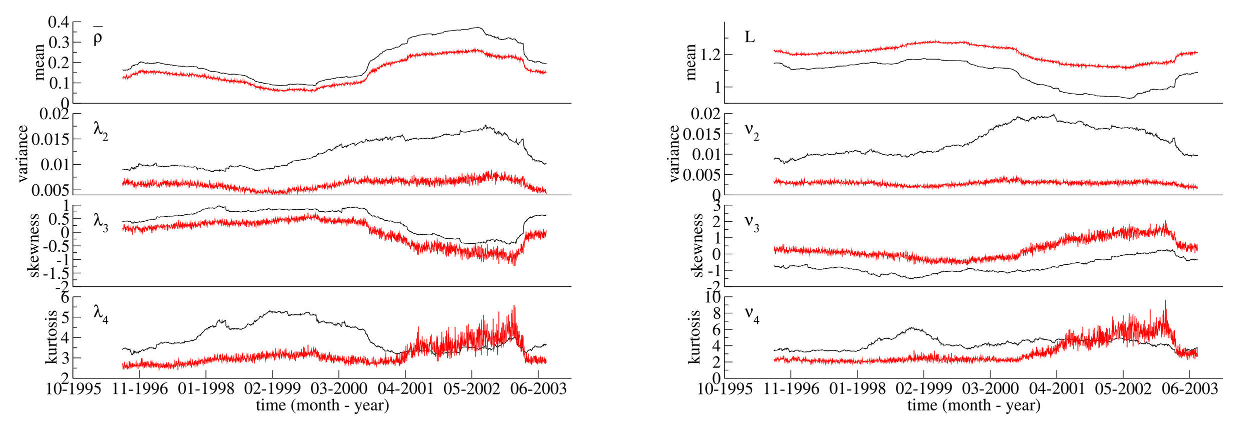

FIGURE 3: Moments for correlations (left) and distances (right). Here, we used time windows of 500 days, with a window step length of 1 day. Black line for real time series and red line for random time series.

As shown in [2, 5], we can form conclusions about the correlation between moments. The mean and variance of the correlation coefficients are highly correlated (0.779), the skewness and kurtosis are also correlated and the mean and skewness are anti-correlated. For the moments of the distances other conclusions can be made. The mean and the variance of the distances of the tree are anti-correlated but the skewness and the mean continue to be anti-correlated. The mean of the correlation coefficients and mean of the distances of the tree are anti-correlated, as expected from the equation of the distances.
The same study of the variation in time of the moments can be made for the random time series (red lines in Figure 3).Some of our previous conclusions maintain, as for the moments of the correlations, the relation between mean and variance and between mean and skewness and for the distances in the MST, the relation between mean and variance and between mean and skewness. But now we can see that the skewness and the kurtosis of the correlation coefficients are not correlated as before. And new relations seem to appear: the kurtosis of the correlation and distances are correlated; the skewness of the correlations and distances are anti-correlated.

5. CONCLUSIONS

The MST for Random time series are very different from the Real MST, but the dependence on time is very similar. Removing the crisis points where extreme events not characterized by a Gaussian distribution occur, the dependence on time is well described.

Acknowledgments

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