

Minimal Spanning Trees (MST) analysis of random time series from different distributions

Ricardo Coelho

School of Physics
Trinity College Dublin

Porto, October 2006

Outline

- 1 Introduction
- 2 Definitions
 - Price
 - Logarithmic Return
 - Correlations
 - Distances
- 3 Minimal Spanning Trees
 - Real MST
 - Random MST
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 - Distances
- 4 Summary

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Introduction

Econophysics group of the Trinity College Dublin:

- Prof. Peter Richmond
- Dr. Stefan Hutzler
- Dr. Przemyslaw Repetowicz

Collaborations:

- Prof. Brian Lucey (Business School of Trinity College Dublin)
- Prof. Claire Gilmore (McGowan School of Business of King's College, Pennsylvania)

Part of this work can be found in the Physics arXiv:

`physics/0601189`

`physics/0607022`

We studied two different sets of data.

- A portfolio from the London Stock Exchange FTSE100 index:
 - 67 stocks
 - Daily closing price
 - From 2nd August 1996 until 27th June 2005
- Different World Indices
 - 53 countries' equity markets
 - Wednesday closing price (weekly returns)
 - From 8th January 1997 until 1st February 2006

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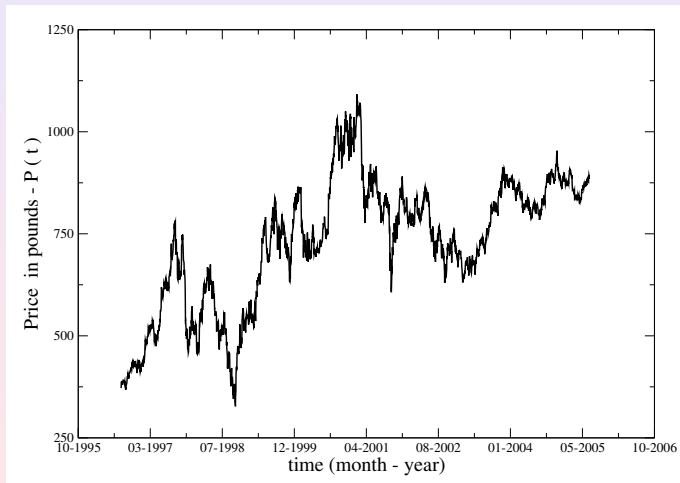
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Price of a stock from the FTSE100 (HSBC)

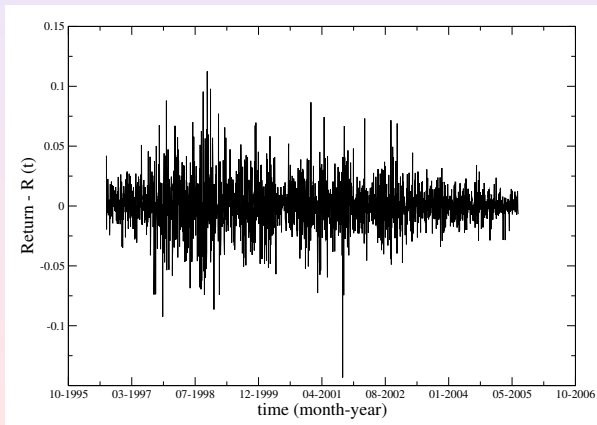


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Log-return of a stock from the FTSE100 (HSBC)

$$R_i(t) = \ln P_i(t) - \ln P_i(t - 1) \quad (1)$$



Normalized return

Subtracting the mean and dividing by the standard deviation:

$$\tilde{\mathbf{R}}_i = \frac{\mathbf{R}_i - \langle \mathbf{R}_i \rangle}{\sqrt{\langle \mathbf{R}_i^2 \rangle - \langle \mathbf{R}_i \rangle^2}} \quad (2)$$

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Correlation between stocks i and j - $\tilde{\mathbf{R}}_i \cdot \tilde{\mathbf{R}}_j$

$$\rho_{ij} = \frac{\langle \mathbf{R}_i \mathbf{R}_j \rangle - \langle \mathbf{R}_i \rangle \langle \mathbf{R}_j \rangle}{\sqrt{(\langle \mathbf{R}_i^2 \rangle - \langle \mathbf{R}_i \rangle^2) (\langle \mathbf{R}_j^2 \rangle - \langle \mathbf{R}_j \rangle^2)}} \quad (3)$$

- $\langle \dots \rangle$ is an average over time $\frac{1}{T} \sum_{t'=t}^{t+T-1} \dots$, t is the first day, T is the length of our time series.
- $-1 \leq \rho_{ij} \leq 1$.
- ρ_{ij} form a symmetric $N \times N$ matrix. $\rho_{ii} = 1$.

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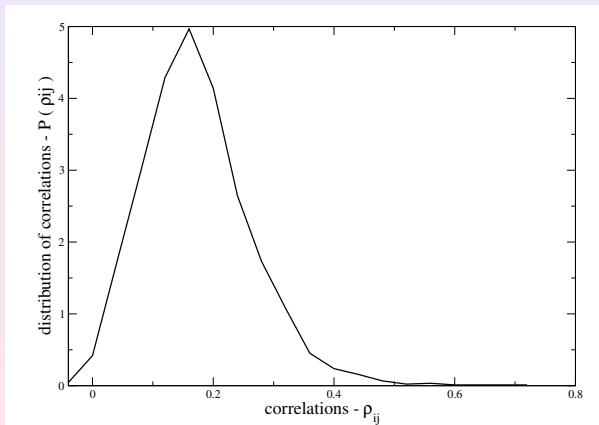
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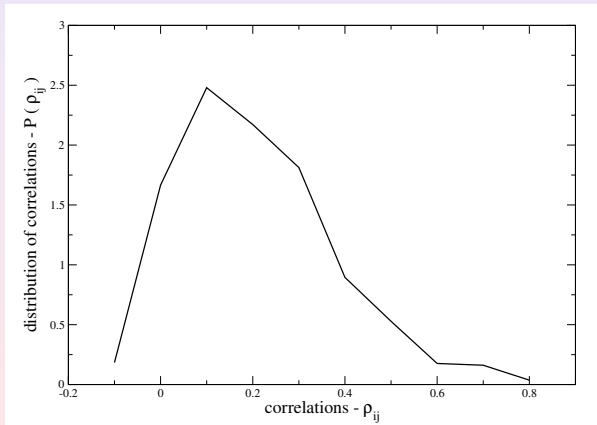
Distribution of the correlations

Full time series, $T = 2322$ days, for the FTSE100 stocks.



Distribution of the correlations

Full time series, $T = 475$ weeks, for the 53 World Indices.



Correlations in time

- With time series of $T = 2322$ days, we can divide in small windows of T' ($< T$) days.
- T' must obey $\frac{N}{T'} < 1$.
- For each window we calculate the correlations.

Moments of the correlations

- Mean

$$\bar{\rho} = \frac{2}{N(N-1)} \sum_{i < j} \rho_{ij} \quad (4)$$

- Variance

$$\lambda_2 = \frac{2}{N(N-1)} \sum_{i < j} (\rho_{ij} - \bar{\rho})^2 \quad (5)$$

- Skewness

$$\lambda_3 = \frac{2}{N(N-1)\lambda_2^{3/2}} \sum_{i < j} (\rho_{ij} - \bar{\rho})^3 \quad (6)$$

- Kurtosis

$$\lambda_4 = \frac{2}{N(N-1)\lambda_2^2} \sum_{i < j} (\rho_{ij} - \bar{\rho})^4 \quad (7)$$

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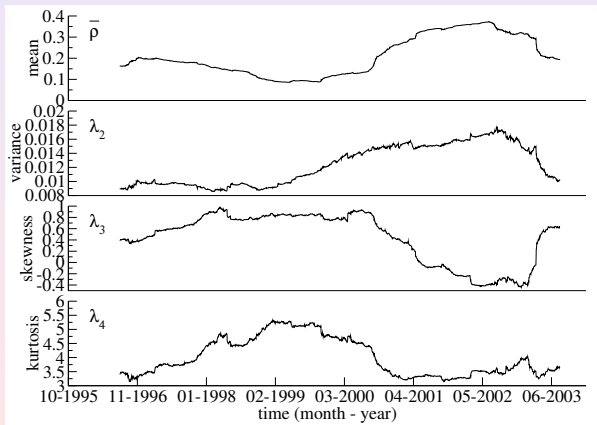
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Correlations and anti-correlations between moments

FTSE100, $T = 500$ days, $\delta T = 1$ day.



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Distance between stocks i and j

Metric distance, introduced by Rosario Mantegna, is determined from the Euclidean distance between vectors:

$$d_{ij} = |\tilde{\mathbf{R}}_i - \tilde{\mathbf{R}}_j| \quad (8)$$

So:

$$d_{ij}^2 = |\tilde{\mathbf{R}}_i - \tilde{\mathbf{R}}_j|^2 = |\tilde{\mathbf{R}}_i|^2 + |\tilde{\mathbf{R}}_j|^2 - 2\tilde{\mathbf{R}}_i \cdot \tilde{\mathbf{R}}_j = 2 - 2\rho_{ij} \quad (9)$$

Distance between stocks i and j

$$d_{ij} = \sqrt{2(1 - \rho_{ij})} \quad (10)$$

- $0 \leq d_{ij} \leq 2$, small values imply strong correlations.
- From distances we construct the Minimal Spanning Tree (MST).
- Analyse the properties of these networks.

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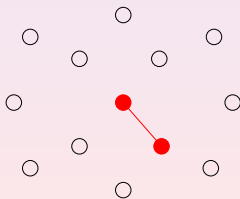
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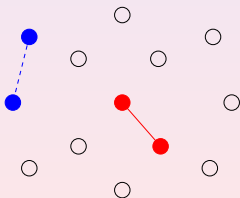
Prim's Algorithm

- Choose the minimum distance between 2 stocks - one link between these 2.
- Find the next minimum distance. If the 2 stocks are not linked or both linked, choose the next minimum distance. If just 1 of the stocks has already links, link them.



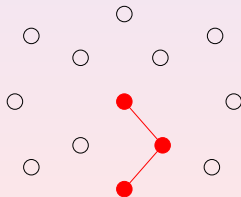
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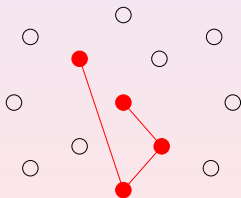
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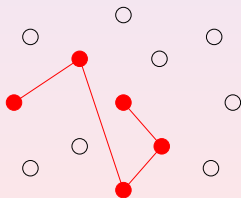
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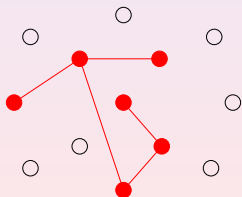
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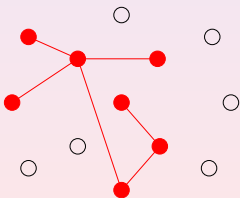
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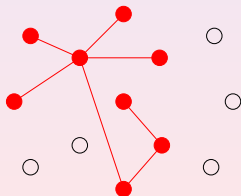
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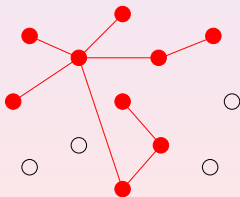
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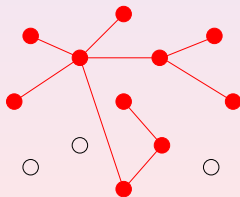
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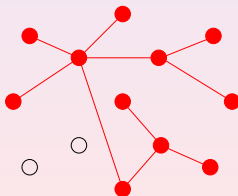
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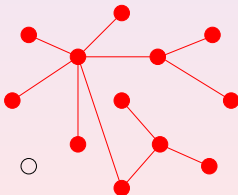
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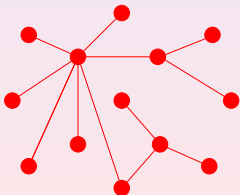
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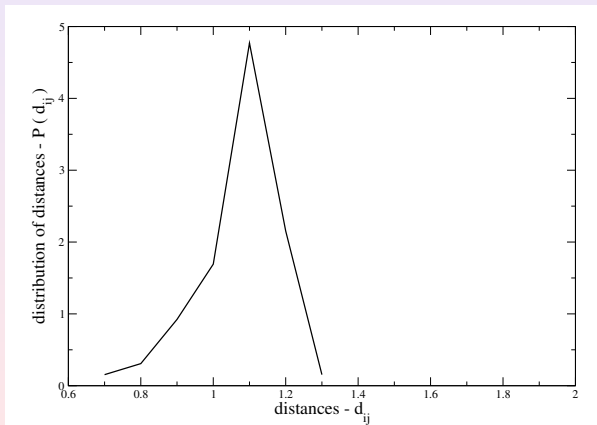
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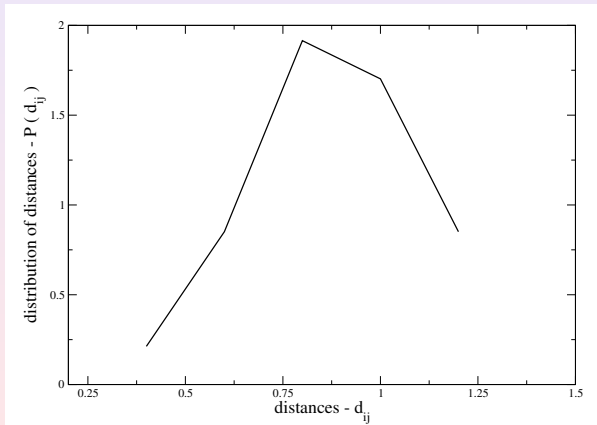
Distribution of the distances

Full time series, $T = 2322$ days, for the FTSE100 stocks.



Distribution of the distances

Full time series, $T = 475$ weeks, for the 53 World Indices.



Moments of the distances

- Mean

$$L = \frac{1}{N-1} \sum_{d_{ij} \in \Theta} d_{ij} \quad (11)$$

- Variance

$$\nu_2 = \frac{1}{N-1} \sum_{d_{ij} \in \Theta} (d_{ij} - L)^2 \quad (12)$$

- Skewness

$$\nu_3 = \frac{1}{(N-1)\nu_2^{3/2}} \sum_{d_{ij} \in \Theta} (d_{ij} - L)^3 \quad (13)$$

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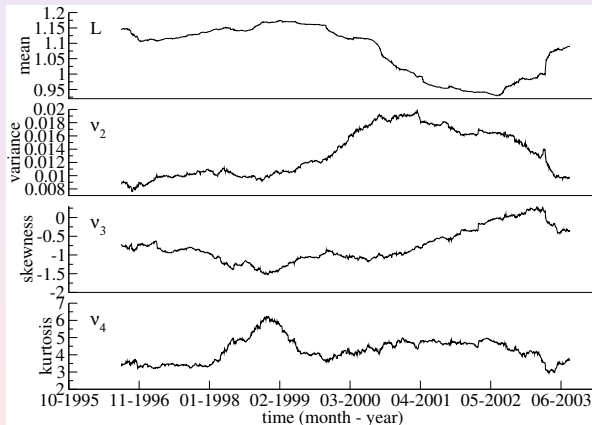
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Correlations and anti-correlations between moments

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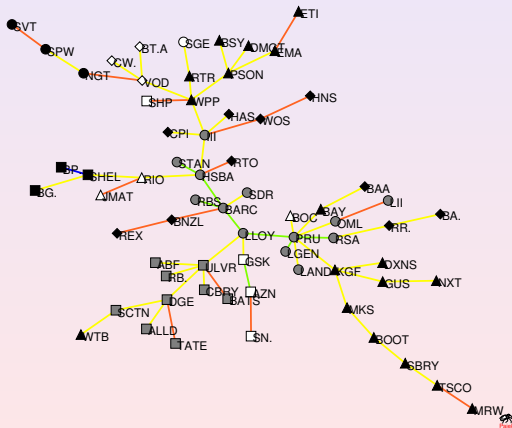


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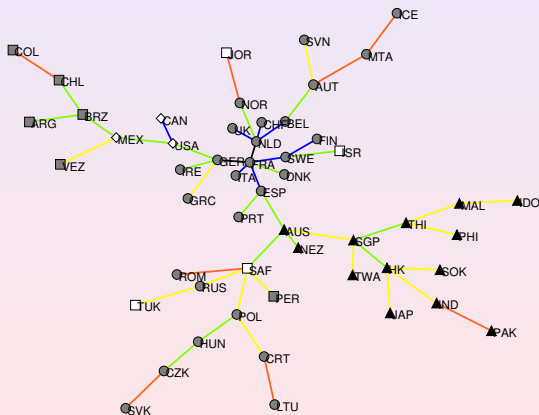
Stocks cluster in industrial sectors (ICB classification)

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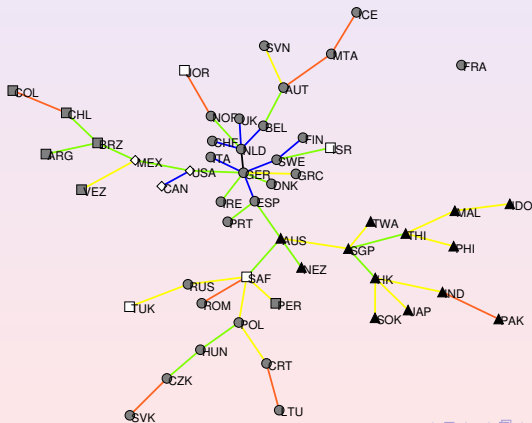
Markets organized by geographical location

Full time series, $T = 475$ weeks, for the 53 World Indices.

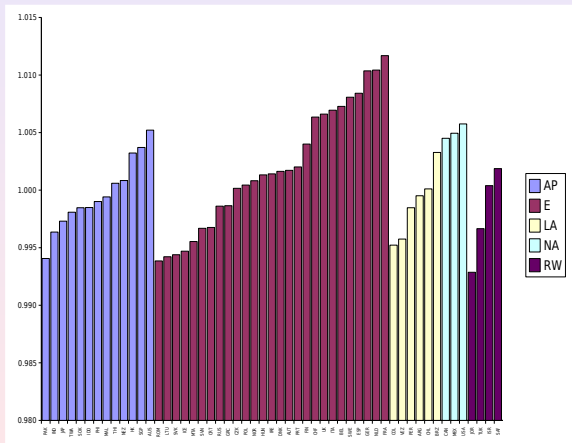


And if we take one country out?

Full time series, $T = 475$ weeks, for the 53 World Indices (without France).



Change in the Mean Length of the MST



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Can we mimic real MST from random data?

- Returns as random variables from a Gaussian distribution

$$r_i(t) = \epsilon_i(t) \quad (15)$$

- Returns as random variables from a T-student distribution

$$r_i(t) = \gamma_i(t) \quad (16)$$

- Returns from a *Market Model* with Gaussian noise

$$r_i(t) = \alpha_i + \beta_i R_m(t) + \epsilon_i(t) \quad (17)$$

- Returns from a *Market Model* with T-student noise

$$r_i(t) = \alpha_i + \beta_i R_m(t) + \gamma_i(t) \quad (18)$$

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Can we mimic real MST from random data?

- Returns as random variables from a Gaussian distribution

$$r_i(t) = \epsilon_i(t) \quad (15)$$

- Returns as random variables from a T-student distribution

$$r_i(t) = \gamma_i(t) \quad (16)$$

- Returns from a *Market Model* with Gaussian noise

$$r_i(t) = \alpha_j + \beta_j R_m(t) + \epsilon_j(t) \quad (17)$$

- Returns from a *Market Model* with T-student noise

$$r_i(t) = \alpha_j + \beta_j R_m(t) + \gamma_j(t) \quad (18)$$

How to fit the Gaussian and T-student distributions?

- Gaussian Distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (19)$$

- T-student Distribution

$$p_k(x) = \frac{\Gamma(k)}{\Gamma(k - \frac{1}{2})} \frac{1}{\sqrt{\pi(2k - 3)}\sigma} \left[1 + \frac{x^2}{\sigma^2(2k - 3)}\right]^{-k} \quad (20)$$

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Parameters for the T-student, from the fractional moments

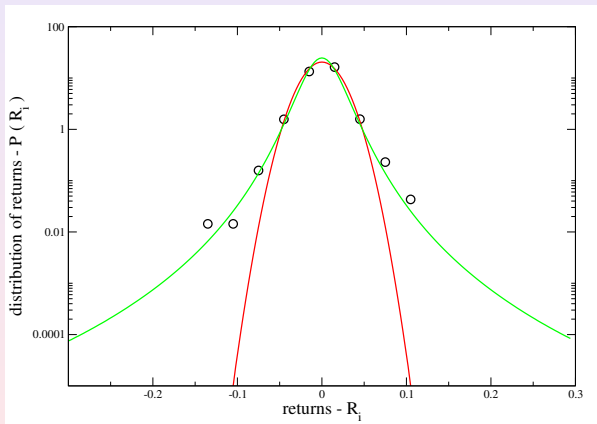
$$\frac{\langle S_N^{\frac{P}{N}-1} \rangle}{\langle S_N^{\frac{P}{N}+1} \rangle} = \frac{1}{(2k-3)\sigma^2} \left[(k-1) \frac{2N}{P} \right] - \frac{1}{(2k-3)\sigma^2} \quad (21)$$

- $\frac{2}{3} < \frac{P}{N} < 1$
- Linear regression give us k and σ^2 :

$$k = \frac{2a-b}{2a} \quad (22)$$

$$\sigma^2 = \frac{1}{a+b} \quad (23)$$

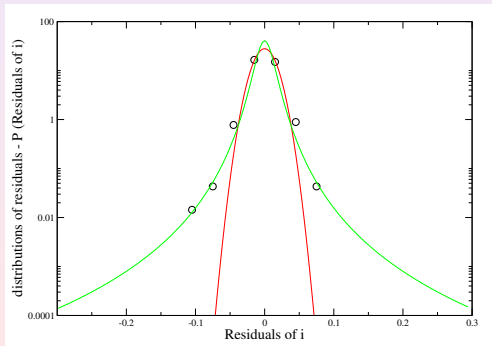
Distribution of the Log-return (HSBC)



Distribution of the Residuals (HSBC)

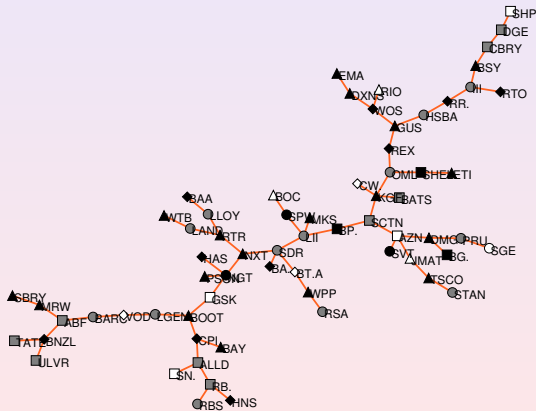
Residuals of a stock is the difference between the returns and the values estimated using the *Market Model*:

$$Res_i(t) = R_i(t) - \alpha_i - \beta_i R_M(t) \quad (24)$$



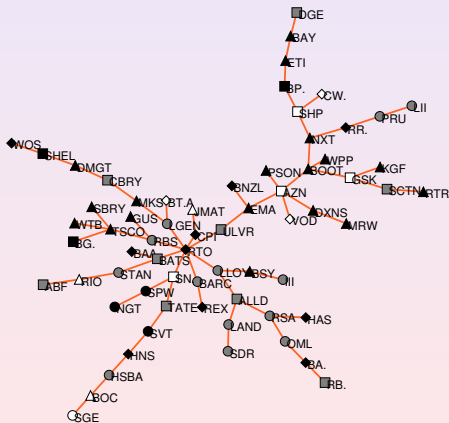
Random Model - Gaussian

Full time series, $T = 2322$ days (pseudo-FTSE100 stocks).



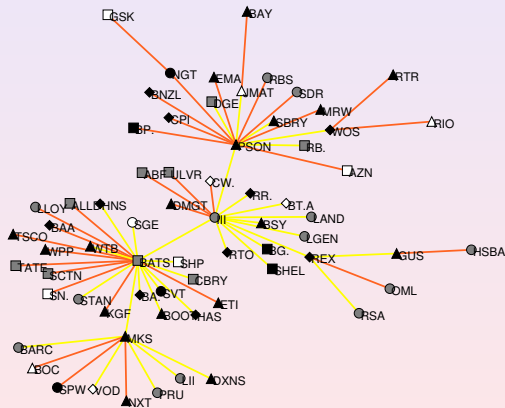
Random Model - T-student

Full time series, $T = 2322$ days (pseudo-FTSE100 stocks).



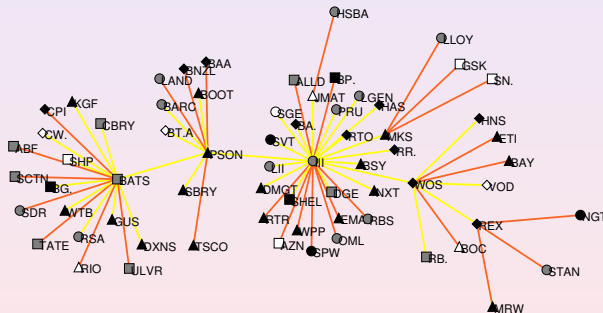
Market Model - Gaussian

Full time series, $T = 2322$ days (pseudo-FTSE100 stocks).



Market Model - T-student

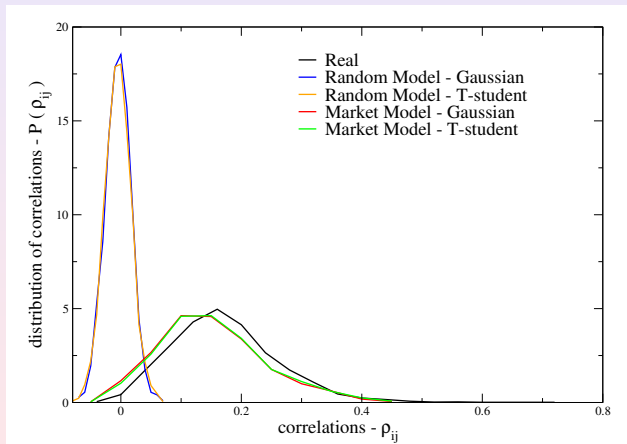
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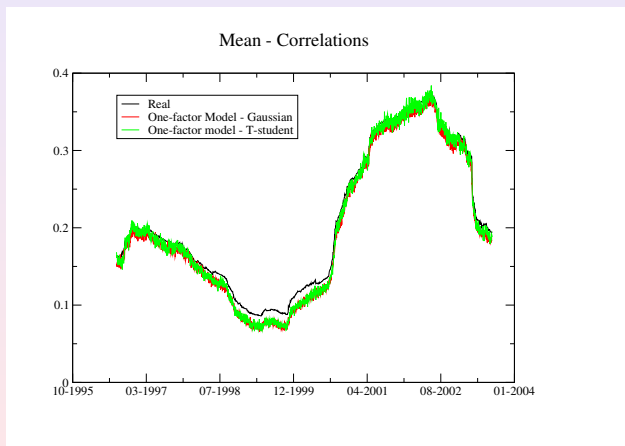
Outline

- 1 Introduction
- 2 Definitions
 - Price
 - Logarithmic Return
 - Correlations
 - Distances
- 3 Minimal Spanning Trees**
 - Real MST
 - Random MST
 - Correlations**
 - Distances
- 4 Summary

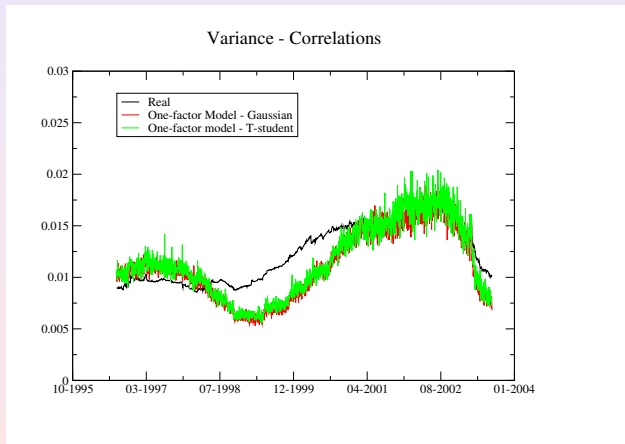
Distribution of correlations



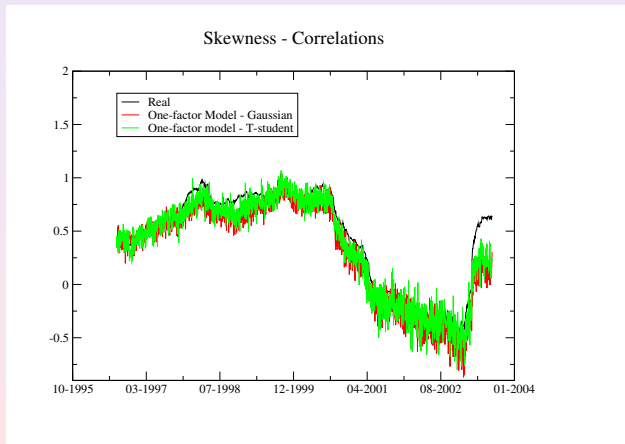
Mean correlation in time



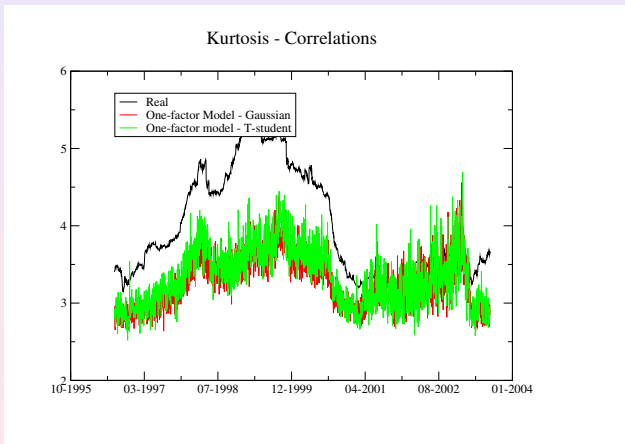
Variance of correlations in time



Skewness of correlations in time



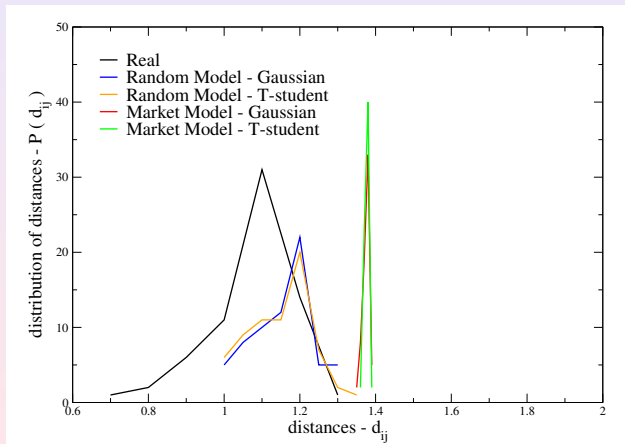
Kurtosis of correlations in time



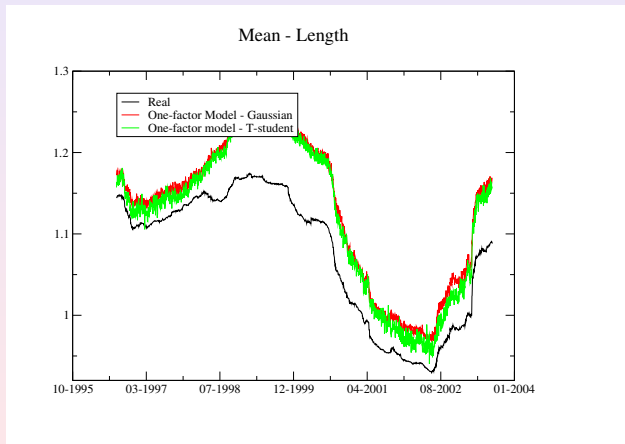
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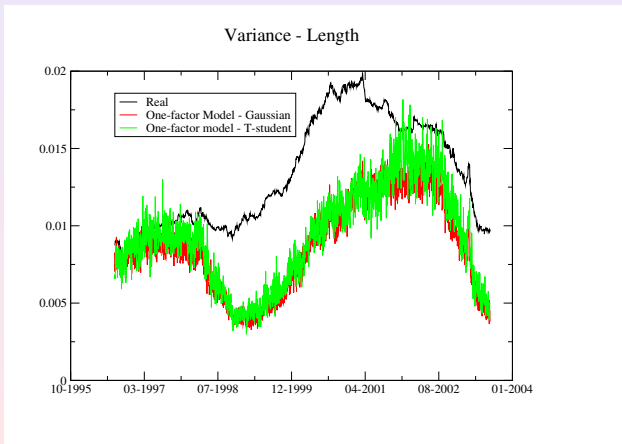
Distribution of distances



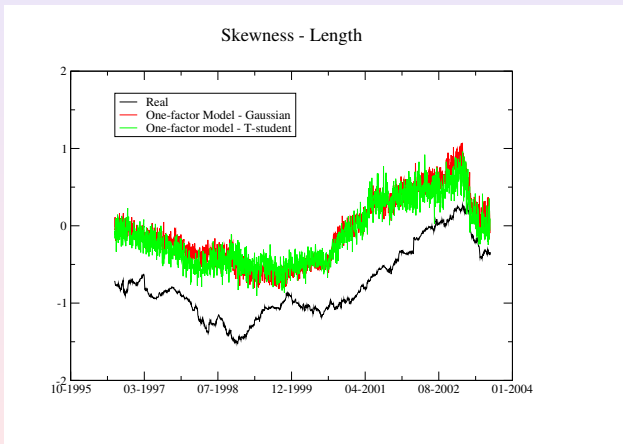
Mean distance in time



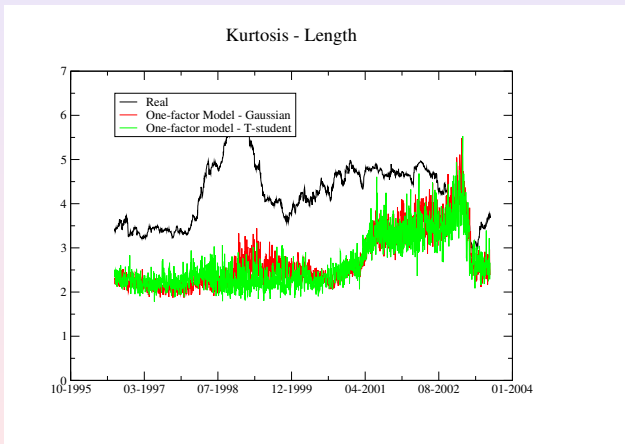
Variance of distances in time



Skewness of distances in time



Kurtosis of distances in time



Conclusions

- Similar behaviour for both the FTSE100 stocks and the World Indices:
 - Mean and variance of correlations are correlated
 - Mean and variance of distances are anti-correlated
 - MST show different clusters (Industrial sectors / Geography)
- We can mimic the moments of correlations from random time series (*Market Model*), compared with the real one.
- The moments of distances and degree (related with the MST) cannot be mimicked with these simple models.
- There is more information in these trees that we should study ...

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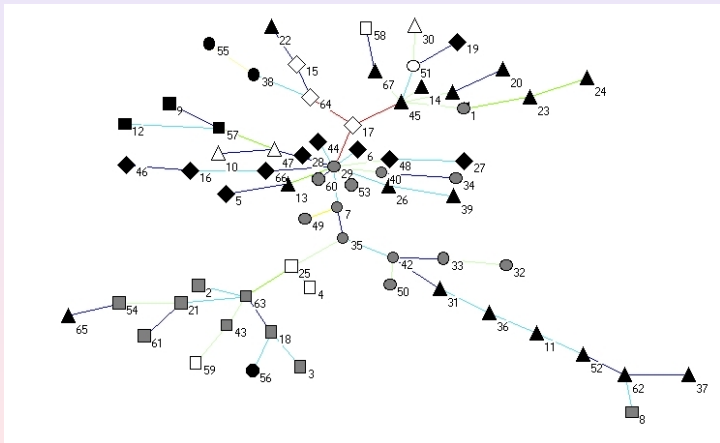
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Future work ... (Earnings per share)



Acknowledgements

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