MA448: K-theory and Solitons
Problem Set 7

Due at 1 pm on Wednesday, 15 April 2009.

1 Monopole Connection

Consider $P(S^2, U(1))$ a $U(1)$ principal bundle over a two-sphere S^2. The base sphere is covered with two charts U_N and U_S defined using the spherical coordinates

$$U_N = \left\{ (\theta, \phi) \,\middle|\, 0 \leq \theta \leq \frac{1}{2}\pi + \epsilon \right\}, \qquad U_S = \left\{ (\theta, \phi) \,\middle|\, \frac{1}{2}\pi - \epsilon \leq \theta \leq \pi \right\}.$$

In some local trivializations the connection is given by

$$A_N = ig(1 - \cos\theta)d\phi, \qquad A_S = -ig(1 + \cos\theta)d\phi. \tag{1}$$

Find the transition function t_{NS} such that the expressions above define a connection on $P(S^2, U(1))$. What are the restrictions on g for this to be the case?

What is the *monopole charge* (also called the *monopole number*) of $P(S^2, U(1))$.

2 Monopole Curvature

Find the curvature of the connection of Eq. (1) in each chart. What is the relation between these expressions in different charts?

3 A Characteristic Class

Find the integral of the curvature two form of the connection of Eq. (1) over the base sphere S^2.

4 Parallel Transport

Consider the equator curve

$$\gamma_1 : [0, 1] \to S^2,$$
$$\gamma_1 : t \mapsto (\theta, \phi) = (\frac{1}{2}\pi, 2\pi t).$$

Since it is a loop, the fiber at $\gamma(0)$ is identical to the fiber at $\gamma(1)$. Using the expression for A_N in Eq. (1) find the parallel transport $\Gamma(\gamma_1)$ action on this fiber.

Do the same analysis for the curve

$$\gamma_2 : [0, 1] \to S^2,$$
$$\gamma_2 : t \mapsto (\theta, \phi) = (\frac{1}{3}\pi, 2\pi t).$$