MA448: K-theory and Solitons
Problem Set 5

Due at 10 am on Thursday, 26 February 2009.

1 External Product in $K^*(X)$

Establish the relation between $S^iX \wedge S^jY$ and $S^{i+j}(X \wedge Y)$.
Demonstrate that $\tilde{K}(S^iX \wedge S^jY) \approx \tilde{K}(S^{i+j}(X \wedge Y))$.

2 Ring Structure on $\tilde{K}^*(X)$.

Given the ring structure on $\tilde{K}^*(X) = \tilde{K}^0(X) \oplus \tilde{K}^1(X)$. Considering the $\tilde{K}^0(X)$ component prove that the external product given by

$$\tilde{K}^*(X) \otimes \tilde{K}^*(X) \to \tilde{K}^*(X \wedge X) \to \tilde{K}^*(X),$$

(where the last map is induced by the diagonal embedding) restricts to the standard product on $\tilde{K}^0(X)$.

3 K Groups

Find K groups and reduced K groups of
a) a point,
b) a two-torus T^2,
c) a sphere S^2,
d) a three-sphere S^3.

4 Ring Structures

Determine the ring structure on the K groups found in question 3.